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INTERMEDIATE ARITHMETIC
CHARCELLON



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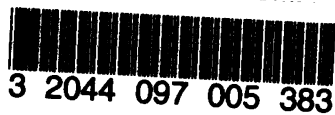
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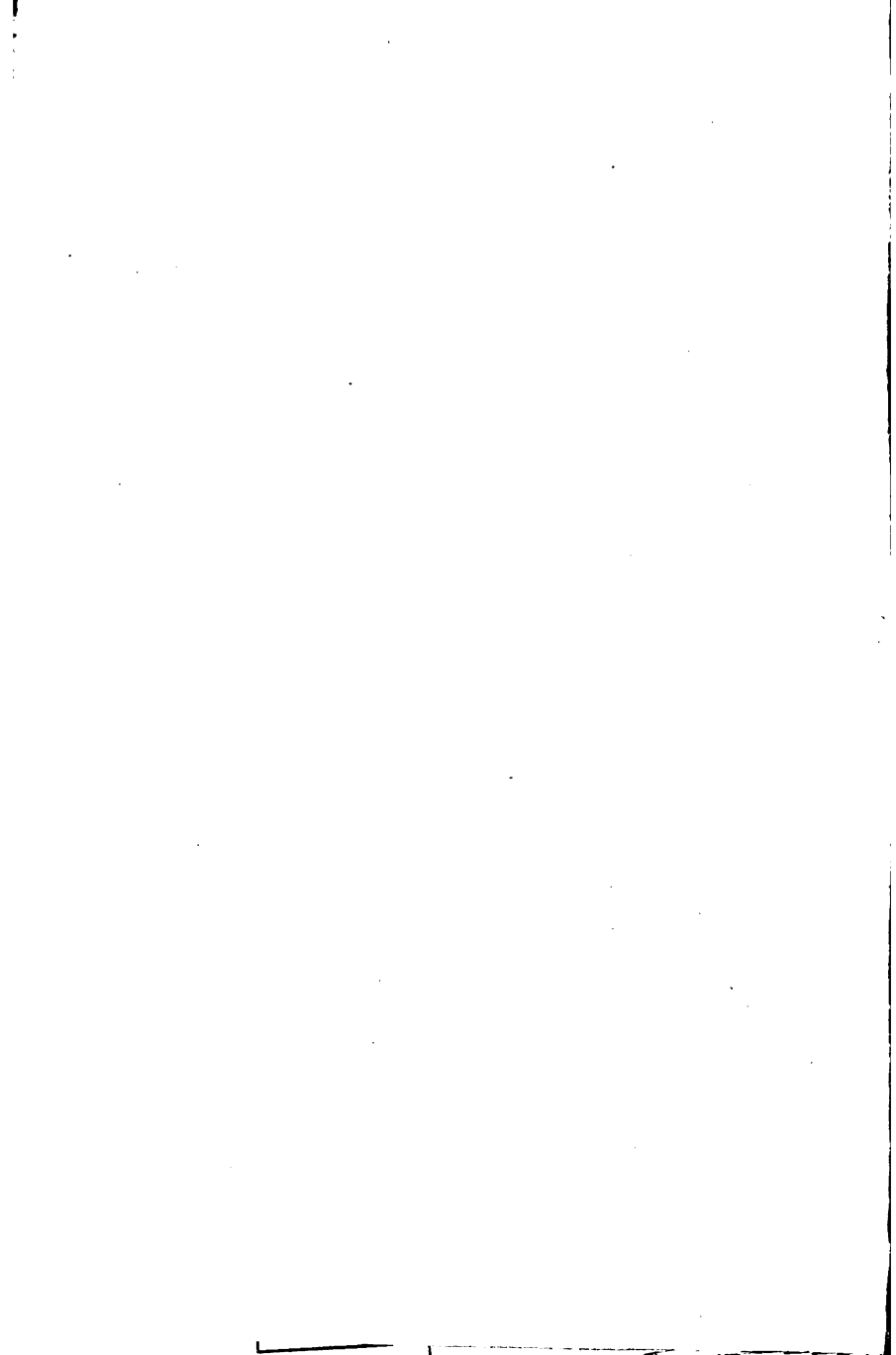
No book shall be allowed to circulate until one month after its receipt.

Received May 15, 1903.

HARVARD
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*ARITHMETIC**GLOBE SERIES IN THREE BOOKS*

INTERMEDIATE ESSENTIAL PRINCIPLES

BY

WILLIAM E. CHANCELLOR, A.M.

SUPERINTENDENT OF SCHOOLS, BLOOMFIELD, N.J.



GLOBE SCHOOL BOOK COMPANY
NEW YORK AND CHICAGO

February 1924

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JANUARY 25, 1924

"The merely oral method does not suffice for the production of the independent scholar. The text-book method holds the pupil responsible for mastering critically the printed page. It requires him to make alive again its thoughts and perceptions. This training in the critical study of the printed page fits him for the work of independent investigation."

W. T. HARRIS, LL.D.,

United States Commissioner of Education.

Address on Horace Mann, N. E. A., Buffalo, 1896.

"At about eight or nine there begins a new period, which for nearly four years constitutes a unique stage of life. . . . Child study suggests very plainly that this period should be devoted mainly to drill, habituation and mechanism. . . . Discipline should be the watchword here."

President G. STANLEY HALL, LL.D.,

Clark University.

From Address on "The Ideal School,"

Council, National Educational Association, 1901.

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M. P. 1

MANHATTAN PRESS
474 WEST BROADWAY
NEW YORK

PREFACE

THIS book is for boys and girls who can use numbers into the thousands in addition and subtraction, who know what ratio is and understand something about fractions, who know some facts of measures, and who have studied the multiplication tables. For these boys and girls there is here about two years' work, chiefly in practice, partly in extension of knowledge.

The new topics introduced here are direct outgrowths of earlier topics, and are presented in their simplest forms and relations. Ratio develops into proportion, and proportion into equation. The common fraction is correlated with percentage, with the decimal, and with ratio and proportion. Factoring is centered upon dividing, and cancellation is centered upon both. The method of long division is explained. A few pages are devoted to each of two topics, greatest common divisor and least common multiple. But the emphasis of the book is upon the fundamental operations of addition and subtraction, multiplication and division for drill, and upon fractions and measures for development of arithmetical knowledge and skill. The advanced topics include the multiplication and division of fractions by fractions and of decimals by decimals, the principles of the fundamental operations and processes, very simple measuring by geometric methods, a series of exercises in United States money, and a brief treatment of simple interest.

This book is more nearly topical in arrangement than the Primary Book of this Series; but an essentially spiral character is secured by frequent reviews. Subjects are not exhausted before we pass to new subjects or return to old subjects displayed in new matter.

It is sound child-psychology to believe that boys and girls in fourth, fifth, and sixth grades have singular power and even pleasure in undergoing the exercises of drill in any and every subject. It is fortunate that there is such a period in the mental and physical life as that passed by children usually in their fourth, fifth, and sixth grades at school. It is unfortunate that sufficient advantage of this age in life has not yet been taken by the text-book writers. In particular, in arithmetic we have many excellent books and monographs discussing methods, devices, and exercises in teaching the elementary facts of number from one to twenty, and very many books discussing and exemplifying arithmetic as a science. Most of these latter books are logical, but few are psychological. They are treatises upon arithmetic or manuals for adult teachers rather than text-books for young students. In a bibliography of many hundred titles one finds not ten books and essays which even try to present, or to demonstrate proper methods of presenting, to boys and girls nine to twelve years of age such fundamental and intermediate operations as are indicated in this book. Our pedagogical philosophers have clear and correct ideas of the minds of six year old children; and our text-book writers have at least scientifically arranged the topics of arithmetic for fifteen year old boys and girls. In all books upon methodology the consideration of this intermediate department is extremely general and superficial. I fail to find any considerable number of authors who have deliberately set before themselves the problems of the minds of the boys and girls who are likely to need such a book as this.

On the other hand, our business men say that the boys and girls who leave school at the ages of twelve to fourteen know nothing accurately in arithmetic, not even the fundamental operations. This is a curious fact since the very power of the ten to twelve year old child is to undergo drill. This book is an effort to present interesting forms of such drill as will develop quickness and accuracy.

It is not meant that we are to question the principle that "the problem is the unit of arithmetic." It is meant that success in the problem is conditioned by mere mechanical facility and correctness in number-manipulation. There ought to be many problems in considerable variety in every arithmetic text-book. Every characteristic problem ought to be thoroughly studied and completely understood, and this as early in the child's life as possible.

I sympathize strongly, as this Series shows, with the ratio-apostles who teach ratio "early and often." I see in ratio the true key to the fraction which is otherwise the child's arithmetical "bugaboo." But I am far from believing that we can profitably thrust many of the complications of percentage and interest, of denominate numbers, even of fractions, into grades lower than those in which they are nowadays usually found in our American common schools. Hence in this second book of the Series designed for children's manuals I have seized as a true educational opportunity the time of teachers and students for many exercises in the simple, ever useful principles of addition, subtraction, multiplication, and division, and in the elements of common fractions, ratio, proportion, percentage, and decimals. It has often been said that "science is exact measurement." These principles are the instrumentalities of exact measurement, whether in the bookkeeping of a commercial business, or in the speculations of mathematical astronomy, or in the researches of quantitative chemistry.

In work in such a grade as this book is designed to serve it is unquestionable that the boys and girls need a text-book to study. It is very easy for teachers to over-develop the arithmetical instruction of boys and girls, one of whose greatest joys is, if they have been rightly taught, in finding out things for themselves and in setting their own tasks. We are here at a stage in advance of that in which it is constantly necessary to renew in the pupil's mind the sense of the facts that figures are merely the symbols of numbers and that numbers always relate to real things. We can now deal with figures,

manipulating them usually without consciousness of what they represent, in exactly the spirit of the bank employee who forgets that the coins and bank notes he handles are money with purchasing power, and treats his cash, as he ought, in isolation from its concrete values.

It may be argued that this is the disciplinary or cultural view of arithmetic rather than the utilitarian. If so, I hold it because of my understanding of the actual powers and interests of children. The supremacy of the human creature is due to his long infancy, to his long preparation before the responsibilities of self-dependence roll in upon him. In the light of this fact, it is both necessary and desirable to postpone, to the years when the child can understand them, those utilities of education which involve the minute details of practical life. The greatest of all the world's utilities is an affirmative, honest, accurate human mind. Arithmetic, even though chiefly disciplinary, is perfectly justified in its forms and methods if it tends to produce an active, clear, and careful mind.

Author and publishers desire to acknowledge the very helpful suggestions of Superintendent F. E. Spaulding, Ph.D., of Passaic, N.J., in reviewing proofs of these pages.

W. E. C.

BLOOMFIELD, N.J.,
February 22, 1902.

SUGGESTIONS TO TEACHERS

1. The preface explains the general purpose of the book.
2. Read the book itself. The purposes of certain special features appear only when considered in relation to other features.
3. Read also Book I of this Series, which shows what pupils of this grade may be expected to know, or at least to have studied. Whenever the pupils of a class seem weak in matters necessarily preliminary to the grade in which they are, it is essential to review these matters both in instruction and in the pupils' own work. But never expect a class as a whole to know accurately and thoroughly every topic in any subject. Aim for perfect knowledge and proficiency, and one will secure surprisingly good results from some pupils. It is easy to waste time in undue drill of the whole class or in undue attention to individuals. It is very easy in arithmetic to spend an unnecessary amount upon single topics. A book upon the spiral plan, but which does not neglect definite study of each topic as it is introduced or renewed, affords at frequent intervals an opportunity to find out the facts as to the conditions of the class and of its individual pupils.
4. It is unwise to ask children always to solve examples by the method of the book or by that of the teacher. Often children see through problems in ways distinctly individual. If their solutions are logical, correct, clear, and brief, we ought not only to accept, but also to welcome them.
5. One kind of exercises is more valuable than any other in arithmetic: setting by pupils of their own problems and finding the solutions. Exactly as we encourage in our language courses writing by the pupils themselves of various

kinds of compositions, so ought we to encourage the making of their own examples. Such pages as are given here for drill in addition, subtraction, multiplication, and division are merely illustrations of exercises which may be set by the pupils for themselves. Every week there should be a period in the program for invention of problems by the pupils. One caution is to be observed. It is best for the teacher to inspect the problems to see that they are capable of solution. This invention of problems makes the pupils independent of printed answers and stimulates them to try proofs for themselves.

6. By the time the class reaches the middle of this book the multiplication tables should be known exactly, and such facts of the tables of weights and measures as have been introduced should be known systematically in tabular order. It is undesirable to pay much attention to the words of arithmetical rules and definitions at this stage lest the forms conceal the principles. Reasoning is the soul of arithmetic. Memory itself is simply an established process of thinking.

7. Incidentally facts of the business world, as the children see it for themselves, may well be used. What facts will depend upon the neighborhood of the school and of the children's homes. These facts may be used as the bases of their invention of problems. This book uses merely standard facts such as may be seen anywhere. Continued observation of local facts will result in the accumulation of great stores of knowledge for use from time to time in instruction and drill. Among the facts that ought to be known by children and are valuable as material for making problems are such as these; viz.:

(a) Railroad fares are usually three cents per mile. Children pay half fare. What are the costs for familiar distances? for "excursion" and "return" tickets? to great centers of population?

(b) Street-car fares are usually five cents. Children pay three cents. How many miles may one ride for one fare?

Are several tickets sold at lower rates? Are special school children's tickets sold?

(c) Make market lists of prices for provisions, meats, groceries, shoes, dry goods, clothes, carpets, furniture, etc. Place these lists on the blackboard, and use them both in written and in oral work.

(d) Take imaginary trips, singly or in parties, paying all expenses, — fares, food, purchases, admissions to places of amusement or recreation, etc.

(e) As far as possible, get the material from the children, but have it accurate.

8. It is unquestionable that a great variety of problems and exercises tends to make the principles themselves very clear. But avoid too much consecutive repetition of processes. We can secure the memory of processes best by variety of associations. Hence, though we ought to make each topic clear at its first presentation, we ought not to dwell too long upon it, but to go forward to the next, returning occasionally to refresh and to enrich our former knowledge.

9. Neatness in writing tends to accuracy in all exercises. Let us encourage excellent work with pen, pencil, or chalk crayon by commending it. Poor work may be due to defective eyesight, which causes increasing trouble with every added year of life, or the eyes may not be in focus with each other. Blackboard work ought to be clear and large. The figures written by most school children are too small and cramped.

10. In working through such a book as this it is well for a teacher to remember that in the early part of a term or year some problems may prove too hard for the pupils that they will be able to solve easily later in the term or in the year. If the problems are too easy, it is a simple matter to increase the difficulties by adding another step or by dealing with larger figures. While the class is studying the topics, the teacher is studying the class; and while the class improves in industry and ability and technical knowledge, the teacher improves in instruction and in assignment of work through

greater familiarity with the needs and powers of the individuals in the class.

11. In the practical use of this book a greater or a less proportion of the problems will be found too difficult for even the best pupils upon the first consideration of them.

(a) Sometimes it will be found advisable to give oral instruction regarding a problem.

(b) Many of the class will be able to return a few weeks later to a hitherto unsolved problem with power to solve it unaided by the teacher.

(c) Where pupils fail upon many problems, the failure may be due to inability properly to reason out in the imagination both the processes and their concrete bases in ratio and counting. Such pupils need not always more teaching in elementary number and measurement, but more often exercising of their own mental activity by methods suited to their present age in this grade.

A book in which every problem is easy for nearly every child is too easy to give proper inducement for earnest, steady upbuilding effort on the part of its students.

12. The plan here has been to make a systematic outline, leaving to the teacher's judgment of the daily needs of the class exactly the number of problems and upon exactly what pages they should be for the ever insistent "next lesson." Something new for curiosity and a good deal that is old for power and for skill make the right combination for lessons in any subject. From a half page to a page will be usually a reasonable lesson, including both class and home work.

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FACTS FOR PROBLEMS

A brick is 8 in. $\times$ 4 in. $\times$ 2 in. A flatiron weighs 6 lb.

1 M. shingles, laid 4 in. to the weather, cover 100 sq. ft.

A lath is 4 ft. $\times$ $1\frac{1}{2}$ in. A clapboard is 4 ft. $\times$ 6 in.

A roll of wall paper is 24 ft. $\times$ 18 in.

A cubic foot of water weighs 1000 oz.; of ice, 930 oz.

Passenger trains move at the rate of 40 mi. per hour.

Fast mail trains can make a mile a minute.

The greatest speed of the trotting horse is a mile in two minutes.

At sea level the weight of air upon 1 sq. in. of surface is 14.7 lb.

The cost of life insurance in "old-line" companies per \$1000 is:
At 26 yr. of age, \$21; at 38 yr., \$30; at 50 yr., \$48.50.

Letters are 2 ¢ per oz. and each fraction of ounce.

Newspapers and periodicals, when mailed by publishers, cost 1 ¢ per lb. In the ordinary course of the mail they cost 1 ¢ per 4 oz.

Printed matter, unsealed, costs 1 ¢ per oz.

Merchandise costs 1 ¢ per oz.

Telegrams cost 25 ¢ for 10 words, with 2 ¢ for each additional word, for distances within the local telegraph district; within the next "zone," 40 ¢ for 10 words; etc.

A dog weighs 40 lb.; a man, 150 lb.; a horse, 1000 lb.; an elephant, 8000 lb.; a locomotive, 150,000 lb.; an ocean steamship, 20,000,000 lb.

Sound is a radiating energy, causing waves that travel 1080 ft. per sec. in the air and 4700 ft. per sec. in the ocean.

Light is also a radiating energy, whose waves travel 196,000 mi. per sec. through the ether.

Sun time varies 1 hr. in 800 miles, latitude of New York City.

TABLES

COUNTING MEASURE

12 units = 1 dozen	12 = 1 doz.
12 dozen = 1 gross	12 doz. = 1 gr.
12 gross = 1 great gross	12 gr. = 1 G. gr.
20 units = 1 score	20 = 1 sc.

SQUARE MEASURE

144 square inches = 1 square foot	144 sq. in. = 1 sq. ft.
9 square feet = 1 square yard	9 sq. ft. = 1 sq. yd.
$30\frac{1}{4}$ square yards = 1 square rod	$30\frac{1}{4}$ sq. yd. = 1 sq. rd.
160 square rods = 1 acre	160 sq. rd. = 1 A.
43,560 square feet = 1 acre	43,560 sq. ft. = 1 A.
640 acres = 1 square mile or section	

CUBIC MEASURE

1728 cubic inches = 1 cubic foot	1728 cu. in. = 1 cu. ft.
27 cubic feet = 1 cubic yard	27 cu. ft. = 1 cu. yd.

CORD MEASURE

16 cubic feet = 1 cord foot	16 cu. ft. = 1 cd. ft.
128 cubic feet = 1 cord	128 cu. ft. = 1 cd.

TIME MEASURE

60 seconds = 1 minute	60 sec. = 1 min.
60 minutes = 1 hour	60 min. = 1 hr.
24 hours = 1 day	24 hr. = 1 da.
7 days = 1 week	7 da. = 1 wk.
12 months = 1 year	12 mo. = 1 yr.
30 days count as 1 month usually	30 da. = 1 mo.
365 days count as 1 year usually	365 da. = 1 yr.

TABLES

LIQUID MEASURE

4 gills = 1 pint	4 gi. = 1 pt.
2 pints = 1 quart	2 pt. = 1 qt.
4 quarts = 1 gallon	4 qt. = 1 gal.

DRY MEASURE

2 pints = 1 quart	2 pt. = 1 qt.
8 quarts = 1 peck	8 qt. = 1 pk.
4 pecks = 1 bushel	4 pk. = 1 bu.

LINEAR MEASURE

12 inches = 1 foot	12 in. = 1 ft.
3 feet = 1 yard	3 ft. = 1 yd.
$5\frac{1}{2}$ yards = 1 rod	$5\frac{1}{2}$ yd. = 1 rd.
$16\frac{1}{2}$ feet = 1 rod	$16\frac{1}{2}$ ft. = 1 rd.
320 rods = 1 mile	320 rd. = 1 mi.
5280 feet = 1 mile	5280 ft. = 1 mi.

AVOIRDUPOIS WEIGHT

16 ounces = 1 pound	16 oz. = 1 lb.
100 pounds = 1 hundredweight	100 lb. = 1 cwt.
2000 pounds = 1 ton	2000 lb. = 1 T.

2240 pounds = 1 long ton (used for weighing coal at the mines and for merchandise at United States Custom Houses).

CUSTOMARY WEIGHTS

1 bushel of potatoes =	60 pounds
1 bushel of wheat =	60 pounds
1 barrel of flour =	196 pounds

CIRCULAR MEASURE

60 seconds = 1 minute	60'' = 1'
60 minutes = 1 degree	60' = 1°
90 degrees = 1 quadrant	90° = 1 quad.
360 degrees = 1 circumference	360° = 1 circ.

Room 16, Liberty School. Geo. Windom.

1. Multiply 34601 by 385 and prove the answer.

$ \begin{array}{r} 34601 \\ \times 385 \\ \hline 173005 \\ 276808 \\ 103803 \\ \hline 13321385 \end{array} $	$ \begin{array}{r} 385 \overline{) 13321385} \quad (34601 \\ \underline{1155} \\ 1771 \\ \underline{1540} \\ 2313 \\ \underline{2310} \\ 385 \\ \underline{385} \\ 0 \end{array} $	Answer 13321385
---	---	-------------------------------

2 What is the ratio of two years to four months?

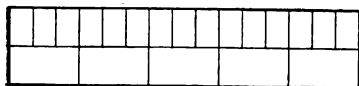
$$1 \text{ yr.} = 12 \text{ mo.}$$

$$2 \text{ yr.} = 12 \text{ mo.} \times 2 = 24 \text{ mo.}$$

$$4 \text{ mo.} : 24 \text{ mo.} = 1 : x \quad 4x = 24 \quad x = 6$$

Answer: The ratio is six.

3 Show by a drawing that $\frac{1}{5} = \frac{3}{15}$



$$\begin{array}{l}
 15 \text{ths.} \quad \frac{3}{15} \\
 5 \text{ths.} \quad \frac{1}{5}
 \end{array}$$

BLACKBOARD

With thumb and fingers, hold the chalk crayon partly under the palm of the hand, and use free arm movements only. It is hard to see figures on the blackboard unless they are at least two inches high, and are written very clear and white.



Write on the blackboard these numbers and add by columns and rows :

1. 1246	9223	6114	4645	8706	3139
2. 2105	4456	8000	9879	5371	2429
3. 4308	2798	7657	2986	1324	4869
4. 6756	7963	2786	4324	7542	2471

DOLLARS AND CENTS

\$ is the sign for dollars. \$5 \$8 \$2

¢ is the sign for cents. 30¢ 25¢ 75¢

We do not write five dollars and thirty cents, using the signs for both dollars and cents, but the sign for dollars only with a period . called the **decimal point**. \$5.30, \$8.25, \$2.75.

The decimal point is always placed after the number of dollars and before the number of cents.

Twenty-one dollars forty cents	\$21.40
Sixty-two dollars ten cents	62.10
Thirty-four dollars seventy cents . . .	34.70
	<u>\$118.20</u>

Let us add these :

100¢ = \$1. The cents here make altogether 120¢.

120¢ = \$1 + 20¢ over = \$1.20.

1.	2.	3.	4.	5.
Add \$3.25	\$2.60	\$ 3.10	\$ 4.25	\$13.22
2.45	4.20	20.35	13.75	.51
<u>3.61</u>	<u>5.55</u>	<u>6.70</u>	<u>19.00</u>	<u>17.54</u>

When we add dollars and cents together, we must be very careful to add the units of cents together and the tens of cents together, and the units of dollars and the tens of dollars together. Do you see that the units of dollars are the hundreds of cents?

200¢ = \$? 300¢ = \$? 500¢ = \$? 800¢ = \$?

6. Add \$3.52, 51¢, and \$7 together. Write in columns.

7. Add \$1, \$4.39, and \$21.50 together. Write \$1.00 for \$1.

8. Change two thousand four hundred cents to dollars.

DOLLARS AND CENTS

1. Add \$2.50, \$1.35, and \$2.45; to their sum add 70¢.

2. Add \$1.20, \$3.20, \$2.05, and \$3.

3. \$1.50 Add and tell why we use
 .05 each of the zeros that you see
 .10 in the four different numbers
 1.00 of dollars and cents.

4. From \$2.40 take \$1.30.

 0¢ less 0¢ are 0¢. Write 0 in cents' place.
 \$2.40 4 tens ¢ less 3 tens ¢ are 1 ten ¢.
1.30 \$2 less \$1 is \$1. The *decimal point* in the
 \$1.10 answer divides the ¢ from the \$.

5. Subtract \$2.75 \$3.85 \$12.90 \$8.35 \$6.40
 1.45 1.85 9.85 7.25 .03

6. From \$2.45 take \$1.98.

 We cannot take from 5¢ 8¢, but from 4
 \$2.45 tens ¢ we can take 1 ten ¢. 15¢ - 8¢ = 7¢.
1.98 We cannot take 9 tens ¢ from 3 tens ¢, but from
 .47 20 tens ¢ we can take 10 tens ¢.

13 tens - 9 tens = 4 tens. \$1 - \$1 = \$0.

7. Subtract \$3.60 \$2.15 \$10.20 \$20.00 \$32.15
 1.75 .90 3.50 8.75 10.00

8. Mary had three dollars and seventy cents, and spent one dollar and a quarter for a beautiful doll. How much money had she left?

9. Sam had eight dollars, and spent six dollars and forty-five cents for a leather-covered football. How much money had he left?

10. Which is more, a thousand cents or eleven dollars?

READING PROBLEMS

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>
1.	123	142	156	116	111	121	133	145	134	144
	<u>16</u>	<u>12</u>	<u>13</u>	<u>22</u>	<u>27</u>	<u>18</u>	<u>15</u>	<u>14</u>	<u>24</u>	<u>14</u>

We may read **1 a**. Add 123 and 16.

	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
2.	118	124	136	128	137	148	125	132	149	150
	<u>-14</u>	<u>-12</u>	<u>-15</u>	<u>-23</u>	<u>-14</u>	<u>-37</u>	<u>-24</u>	<u>-11</u>	<u>-39</u>	<u>-100</u>

The sign $-$ means that the number after it is to be taken from the number before it. We may read **2 k**.—From 118 take, or subtract, 14.

Read the other problems in **2**, using the word “subtract.”

	1	2	3	4	5	6	7	8	9	10
3.	21	34	42	50	111	93	61	222	1000	2000
	<u>$\times 4$</u>	<u>$\times 2$</u>	<u>$\times 5$</u>	<u>$\times 5$</u>	<u>$\times 3$</u>	<u>$\times 2$</u>	<u>$\times 5$</u>	<u>$\times 4$</u>	<u>$\times 5$</u>	<u>$\times 3$</u>

The sign $\times$ means that the number before it is to be multiplied by the number after it. We may read **3 1**.—21 multiplied by 4 are how many? Or,—Multiply 21 by 4.

Read the other problems in **3**, using the word “multiply.”

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
4.	5) <u>55</u>	6) <u>64</u>	3) <u>21</u>	7) <u>70</u>	6) <u>48</u>	9) <u>27</u>	7) <u>28</u>	8) <u>32</u>

4 A. Divide 55 by 5. Read the other problems.

	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>	<i>O</i>
5.	4) <u>128</u>	3) <u>164</u>	5) <u>155</u>	4) <u>164</u>	3) <u>99</u>	5) <u>500</u>	7) <u>728</u>

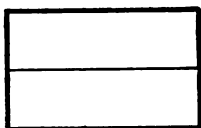
6. $23 \times 3 = ?$ 23 taken 3 times are how many? Or, 23 multiplied by 3 are how many? Or, 3 times 23 is how much?

Read: $25 \times 4 = ?$ $50 \times 2 = ?$ $100 \times 3 = ?$ $33 \times 3 = ?$

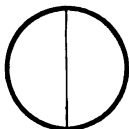
7. $44 \div 2 = ?$ Read: 44 divided by 2 are how many?

Read: $66 \div 3 = ?$ $36 \div 4 = ?$ $48 \div 4 = ?$ $100 \div 10 = ?$

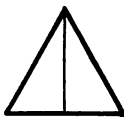
HALVES



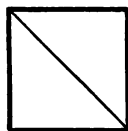
Rectangle in halves



Circle in halves



Triangle in halves



Square in halves

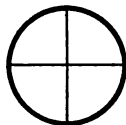
When anything is divided into two equal parts, the parts are called halves.

Draw a line 2 inches long. Divide it into halves.

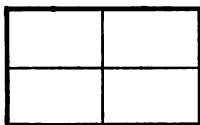
Draw a square. Divide it into halves.

FOURTHS OR QUARTERS

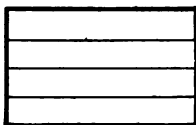
When anything is divided into four parts, each part is called a fourth or a quarter. Four fourths or four quarters make one whole. $4 \times \frac{1}{4} = 1$. $\frac{4}{4} = 1$.



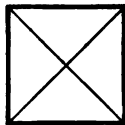
Circle



Rectangle



Rectangle



Square

Into how many parts is the circle divided? each rectangle?

Into how many parts is the square divided?

How many fourths are there in each of these forms?

$\frac{4}{4} = 1$. Four quarters are one. One half of four is 2. $4 \div 2 = 2$. One half of four fourths is two fourths.

$\frac{1}{2}$ of $\frac{4}{4} = \frac{2}{4}$; $\frac{2}{2} = 1$; $\frac{3}{3} = 1$; $\frac{4}{4} = 1$; $\frac{1}{2} = \frac{2}{4}$. Read each of these facts in words.

How much is $\frac{1}{2}$ of $\frac{1}{2}$?

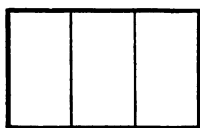
How much are $\frac{1}{2}$ and $\frac{1}{4}$?

THIRDS

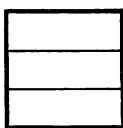
When anything is divided into three parts, each part is called a third. Three thirds equal one whole.



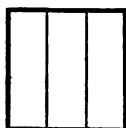
Circle



Rectangle



Square



Square

Into how many parts is the circle divided? the rectangle? each square?

Draw a line three inches long. Divide it into thirds.

One third is written in figures $\frac{1}{3}$.

Two thirds are written in figures $\frac{2}{3}$.

$\frac{3}{3} = 1$. Why is this true? Look and see.

1. $\frac{1}{3}$ of 6 figs = how many figs? There are six times as many thirds in the 6 figs as there are in the one fig.

$$6 \times \frac{1}{3} = \frac{6}{3}. \quad \frac{6}{3} = 2.$$

2. What is $\frac{1}{3}$ of 6? $\frac{1}{3}$ of 9? $\frac{1}{3}$ of 12?

3. Which piece is the larger, $\frac{1}{3}$ of a pie or $\frac{1}{2}$ of a pie?

4. If your mother gave you sixty cents and you spent $\frac{2}{3}$ of it, what amount of money would you have left?

5. Measure the length and width of this page.

6. Measure the length and width of your desk top.

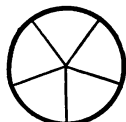
7. How high is the front edge of your desk from the floor?

8. Cut a string twelve inches long. Cut off a piece four inches long. Compare the parts.

9. Cut a strip of paper 6 inches long. Cut another strip 3 inches long. Mark off both into inches.

FIFTHS AND TENTHS

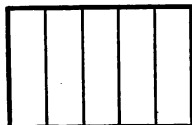
When anything is divided into five equal parts, we call each part one fifth. Five fifths make one whole.



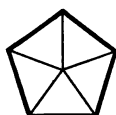
Circle



Square



Rectangle



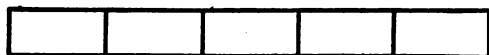
Pentagon



Star



Line



Rectangle

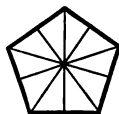
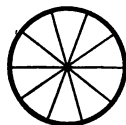
Into how many equal parts is each of these forms divided?

1. Point out two fifths of each of these forms; three fifths; four fifths.

2. Make drawings like these forms on paper, but larger.

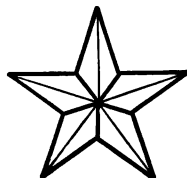
3. Make drawings like these forms on the blackboard.

When anything is divided into tenths, it has ten equal parts. Ten tenths make one whole.



Into how many parts is each of these forms divided? Count and show the parts.

Make drawings like these forms both on paper and on blackboard.



REVIEW

1. A man had 315 horses; he sold 39, but bought 124 more horses. How many horses had he then?

2. In a gathering of 326 men, women, and children, there were 126 men and 150 women. How many children were there?

3. In a school there were 239 pupils, of whom 63 were girls. How many boys were there in the school?

4. Mr. Jones is 84 years old and his son is 59 years old. How much older is the father than the son?

5. Write, in words, the difference between ten thousand fifty and ninety-nine.

6. A coal dealer had 800 tons of coal, of which he sold 333 tons. How many tons had he left?

7. A farmer sold 114 sheep out of a flock of 313. How many sheep had he left?

8. Mr. Smith has 324 sheep and cows, while Mr. Brown has 275 sheep and cows. How many more animals has Mr. Smith than Mr. Brown?

9. A man starts to walk five hundred yards. When he has walked a hundred eighty-two yards, how many yards more has he to walk?

10. Five hundred thirteen children went on a steamboat trip. Three hundred five of the children were boys. How many girls were there?

11. If I have seventy-five dollars, and give away thirty dollars to one man and twenty-eight dollars to another man, how many dollars shall I have left?

12. In one schoolroom there are 34 children, in another room, 40 children, and in a third room, 29 children. Of 150 pupils enrolled how many pupils are absent?

NOTATION

Notation is a method of writing numbers.

The system of notation in general use is called the Arabic Notation. It was introduced into Europe in the twelfth century by the Arabs who founded the empire of the Moors in Spain. It came to Arabia from India.

This notation uses ten characters of figures to express numbers. It is also called a decimal notation, because the numbers increase and decrease in a tenfold ratio.

Figures,	1	2	3	4	5	6	7	8	9	0
Names,	one	two	three	four	five	six	seven	eight	nine	cipher

The last figure, or character, is also called zero, or naught. It has no value when standing alone. The first nine figures are termed significant figures, as each always signifies a value of its own. These figures are also called digits.

We express ten units by using not one figure, but two figures, 10. The cipher occupying the first place, at the right hand, gives a tenfold value to the 1, and shows that there are no units of the first order, or place. The 1 ten is a unit of the second order.

The number 100, equal to 10 tens, is a unit of the third order, and, therefore, the digit 1 occupies the third place. In a decimal system of notation ten units of any order make one unit of the next higher order.

- 1 unit of the first order is expressed by 1.
- 1 unit of the second order is expressed by 10.
- 1 unit of the third order is expressed by 100.
- 1 unit of the fourth order is expressed by 1000.
- 1 unit of the fifth order is expressed by 10000.
- 1 unit of the sixth order is expressed by 100000.

Units of higher orders are written in the same way.

How many units of what order are in 70000000?

NUMERATION

Numeration is a method of reading numbers. In order that numbers may be read conveniently, they are grouped into periods of three orders each.

Learn the names of the periods in the following table :

Hundreds of Quintillions. Tens of Quintillions. Quintillions.	Hundreds of Quadrillions. Tens of Quadrillions. Quadrillions.	Hundreds of Trillions. Tens of Trillions. Trillions.	Hundreds of Billions. Tens of Billions. Billions.	Hundreds of Millions. Tens of Millions. Millions.	Hundreds of Thousands. Tens of Thousands. Thousands.	Hundreds. Tens. Units.
3 6 3,	1 5 4,	7 9 2,	8 8 3,	2 5 1,	3 6 6,	7 2 4.
Period of Quin- tillions.	Period of Quad- rillions.	Period of Tril- lions.	Period of Bil- lions.	Period of Mil- lions.	Period of Thou- sands.	Period of Units.

The periods in their order above Quintillions are Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quattuordecillions, etc., to Vigintillions. Numbers as high as these are seldom needed, except by astronomers in measuring the distances of the stars.

If a number consists of more than one period, the left-hand period may contain one, two, or three figures, but all the other periods must have three figures in each period.

Vacant orders are filled by ciphers.

The system of numeration in use in England has six orders in every period. Hence in the English system a million is a thousand thousand, a billion is a million times a million, a trillion is a million times a billion, and so on.

Point off the number into periods of three figures each, beginning at the right hand, and read each period as if it stood alone, naming its order.

Do not use the word *and* in reading whole numbers.

Ciphers are read only as they show the values of higher figures.

1. Read :

7849630	10000001	309025607	7070000423
341328	20220022	980076011	91234013402
418254	92568987	800309560	91234267109
827171	17643000	987654321	670000000069
6850406	80970503	967004296	1001001001001
5931081	91131140	9738413208	61300400007623
7546738	30180070	5707068080	81008100810081
8080808	800309560	2560530200	100000200993004
48726870634103264	23200300500	1001987000100000	

2. The world has 1,500,000,000 people, and \$1,000,000-000,000 worth of wealth. Write this statement in words.

3. Write in figures the following numbers :

Twenty-eight thousand, three hundred nineteen.

Three million, one hundred forty-five thousand, six hundred eighty-five.

Eighty-six million, four hundred three thousand seven.

Nine hundred nine million, ninety-nine thousand nine.

One hundred forty million, six hundred two thousand seven.

Seven hundred million, twenty thousand.

Five billion, five million, five thousand five.

Sixty trillion, sixty million seventy.

Seventeen trillion, seven million seventy.

Eight billion, seventy million, four hundred thousand, two hundred seventy-six.

DOLLARS AND CENTS

1. Four boys had \$2.40 each. How much money had they in all? We can find this by

MULTIPLICATION

We multiply just as in ordinary multiplication: $0 \times 4 = 0$. $4 \times 4 = 16$. $2 \times 4 = 8$. $8 + 1 = 9$. The period . or *decimal* point we place before the 6 to show that the 9 is \$ and the 60 represents ¢. The boys had each 40¢, but $40¢ \times 4 = 160¢$; and $160¢ = \$1 + 60¢$.

In multiplying money we set the decimal point as many figures to the left as it stood to the left in the multiplicand.

2. Multiply: \$1.75 \$2.25 \$4.13 \$8.15 \$3.98
 3 6 9 12 10

3. The price of each of 7 books was one dollar and twenty-five cents. Mary's mother bought the whole set for her. What change should she receive from a ten-dollar bill?

4. Add: \$8.32 \$1.42 \$8.14 \$10.00 \$14.03
 9.41 3.27 1.90 8.00 .10
 .06 8.29 3.06 9.00 .90
 2.33 .50 2.10 .25 15.00
 8.21 10.00 8.25 1.75 .69

5. Subtract: \$15.00 \$3.33 \$5.00 \$5.00 \$20.00
 8.69 2.98 1.49 3.38 16.75

DOLLARS AND CENTS

1. Six boys had in all \$3.80. They divided the money equally. How much had each? We find this by

DIVISION

$$\begin{array}{r} 6 \overline{) \$3.80} \\ \underline{\$.55} \end{array}$$

6 is found in 3 (hundreds) 0 times.
 6 is found in 33 (tens) 5 times and 3 tens over.
 6 is found in 30 (units) 5 times.

2. Eight girls had in all \$3.60. They went to a store and saw there some dolls at 50¢ each. The clerk told them that he could sell eight dolls at a little lower price each, and gave them the dolls for their money. What price did they pay for each doll?

$$\begin{array}{r} 8 \overline{) \$3.60} \\ \underline{\$.45} \end{array}$$

8 is found in 3 (hundreds) 0 times.
 8 is found in 36 (tens) 4 times and 4 tens over.
 8 is found in 40 units 5 times.

Tell number stories, using the following facts:

3. \$6. and 5 boys buying books : or — : or —.

4. \$2.40 and 12 girls selling violets : or — : or —.

5. 7) \$4.27 6. 9) \$5.49 7. 11) \$13.20 8. 9) \$3.96

REVIEW

WRITE

9. Add 30¢, \$4., \$1.25, \$10., 47¢, and a half dollar.

10. How many inches are there in $7\frac{1}{2}$ ft.?

11. John sold four dozen eggs at 2¢ each egg. What amount did he receive?

12. Mr. Clark's horse weighed $\frac{1}{2}$ ton. How many pounds did he weigh?

13. It is April 1. Mary's baby sister is 1 mo. and 8 days old. What day was the baby's birthday?

MONEY

	1.	2.	3.	4.
Add: \$	432.28	\$ 1.	\$ 400.89	\$ 93.80
	92.01	8.72	398.71	110.
	86.	15.75	.43	364.82
	1491.	18.61	109.61	943.81
	80291.69	92.	83.40	89.62
	<u>3432.86</u>	<u>100.</u>	<u>1298.60</u>	<u>498.</u>

1 represents the chief items in the cost of a schoolhouse, with twenty-four rooms and an assembly hall ; **2** represents the items of the cost of a railroad journey from Boston to Denver, one way ; **3** represents the cost of the stock of a small store of general merchandise ; and **4** represents the money received by a farmer for the chief products of a 320-acre farm in one year.

5. A man who had saved \$10,000 decided to build a house. He estimated its cost in these items: land, \$2000 ; contract for house, \$5500 ; architect's plans and services, \$300 ; heating plant, \$500 ; grading the grounds, \$100 ; furniture, \$1000. When his whole house was finished and his family had moved in, he found that the total cost had been $\frac{1}{5}$ more than his estimate. How much more was the cost than the amount he intended to spend ?

Subtract:

6. \$864.92	7. \$1921.92	8. \$7284391.82
<u>776.15</u>	<u>873.86</u>	<u>5000000.00</u>

9. Multiply six hundred ninety-one thousand four hundred forty-five dollars seventy cents by thirty-two, and prove the result by division.

10. At \$2.50 a day what does a man earn in 300 days ?

UNITS ADDITION

Notice, when adding columns, all combinations making:

(1) 5, 10, 15, or 20; or (2) 9; or (3) 11; or (4) two or three times the same numbers, as 3, 3, 3; or 4, 4, 4.

A	B	C	D ₁	D ₂	D ₃
2 } 5	1 } 9	2 } 11	3 } 10	3 3	3 3
3 } 5	8 } 9	9 } 11	7 } 10	7 }	7 }
6 } 10	7 } 9	3 } 11	6 } 9	6 } 15	6 } 20
4 } 10	2 } 9	8 } 11	2 } 9	2 }	2 }
9 } 15	6 } 9	4 } 11	1 }	1 }	1 }
6 } 15	3 } 9	7 } 11	4 } 11	4 } 5	4 }
7 } 10	5 } 9	5 } 11	7 } 11	7 }	7 7
3 } 10	4 } 9	6 } 11	2 } 10	2 } 9	2 } 10
8 } 15	<u>36</u> <u>36</u>	<u>44</u> <u>44</u>	8 } 10	8 }	8 }
7 } 15			7 7	7 } 15	7 7
8 } 10	F	G	<u>47</u> <u>47</u>	<u>47</u> <u>47</u>	<u>47</u> <u>47</u>
2 } 10	4 } 8	7 2 } 5	Added from above.	Added from below.	Either way.
4 } 5	4 } 8	7 3 } 5			
1 } 5	8 8	7 5 5			
<u>80</u> <u>80</u>	<u>16</u> <u>16</u>	<u>21</u> <u>10</u> <u>10</u>			

Write these rows in columns and add :

I 5, 2, 3, 6, 9, 1, 7, 2, 1, 4, 3, 8, 7, 5, 8.

J 1, 9, 8, 7, 2, 4, 6, 1, 7, 6, 8, 8, 2, 1, 3.

K 4, 4, 4, 5, 5, 6, 3, 8, 7, 9, 6, 5, 9, 9, 2.

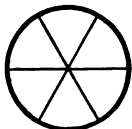
L 9, 9, 9, 6, 2, 5, 5, 6, 4, 3, 3, 3, 1, 7, 8.

M 3, 7, 6, 4, 5, 5, 8, 2, 9, 1, 4, 5, 1, 2, 5.

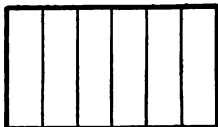
N Add the fifteen columns in I-M above.

SIXTHS AND TWELFTHS

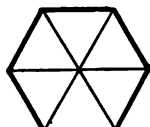
When anything is divided into six equal parts, we call the parts sixths. Six sixths make one whole.



Circle



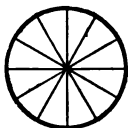
Rectangle



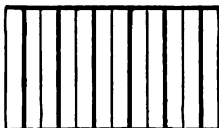
Hexagon

1. Show that each of these forms is divided into halves.
2. Show that each is divided into thirds; into sixths.
3. Make larger drawings of each of these forms on paper; on the blackboard.

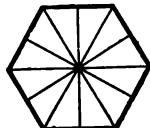
When anything is divided into twelve equal parts, we call the parts twelfths. Twelve twelfths make one whole.



Circle



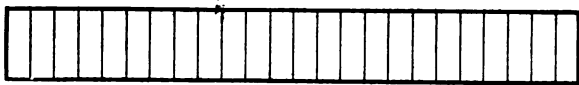
Rectangle



Hexagon

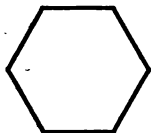
1. Show the various halves in the circle and hexagon.
2. Show thirds of each of these forms.
3. Show fourths of each.
4. Show sixths of each.
5. Make larger drawings of each of these forms on paper; on the blackboard.
6. $\frac{6}{6} = 1$; $\frac{12}{12} = 1$; $\frac{3}{3} = 1$; $\frac{4}{4} = 1$. Read these facts.
7. Which is larger, $\frac{1}{3}$ or $\frac{1}{6}$? Why? $\frac{1}{6}$ or $\frac{1}{12}$? Why?
8. Cut out forms to show the answers to 6 and to 7.
9. Which is larger, $\frac{1}{6}$ or $\frac{2}{12}$? $\frac{2}{6}$ or $\frac{1}{3}$? $\frac{1}{3}$ or $\frac{4}{12}$?

FRACTIONS

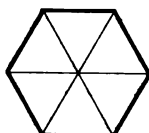


1. Point out $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{24}$ of this oblong.
2. Show that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12} = \frac{12}{24}$.
3. Show that $\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24}$.

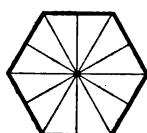
Hexagon



Sixths

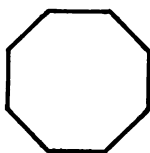


Twelfths

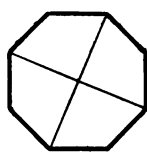


4. Show that $\frac{1}{6} = \frac{2}{12}$, $\frac{2}{6} = \frac{4}{12}$, $\frac{3}{6} = \frac{6}{12}$.

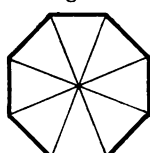
Octagon



Fourths

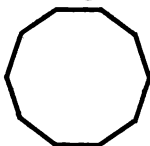


Eighths

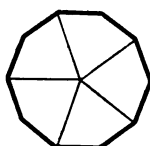


5. Show that $\frac{1}{4} = \frac{2}{8}$, $\frac{2}{4} = \frac{4}{8}$, $\frac{3}{4} = \frac{6}{8}$.

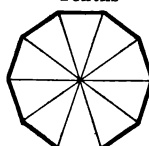
Decagon



Fifths



Tenths



6. Show that $\frac{1}{5} = \frac{2}{10}$, $\frac{2}{5} = \frac{4}{10}$, $\frac{3}{5} = \frac{6}{10}$.

THE FRACTION

A **unit** is one, a whole thing.

A **fraction** is one or more of the equal parts of a unit.

A **common fraction** is expressed by two numbers, one number written below the other, with a line between them.

Thus $\frac{7}{8}$, $\frac{3}{10}$, $\frac{1}{2}$, are common fractions.

The **terms** of a fraction are the **numerator** and the **denominator**.

The number below the line is called the **denominator**. It tells into how many equal parts the unit has been divided. The greater the number of parts, the smaller is each part. $\frac{3}{8}$ is less than $\frac{3}{7}$. $\frac{12}{20}$ is greater than $\frac{12}{40}$. $\frac{1}{4}$ of an orange is more than $\frac{1}{6}$ of the orange. Show by drawings.

The number above the line is called the **numerator**. It tells the number of equal parts taken.

$$\begin{array}{c} \text{numerator} \quad 5 \\ \hline \text{denominator} \quad 6 \end{array}$$

A **proper fraction** is a fraction whose value is less than one. Its numerator is less than its denominator; $\frac{2}{3}$, $\frac{7}{9}$, $\frac{13}{14}$, $\frac{8}{9}$.

An **improper fraction** is a fraction whose value is equal to or greater than one. Its numerator is equal to or greater than its denominator; $\frac{4}{4}$, $\frac{7}{7}$, $\frac{12}{8}$, $\frac{21}{10}$.

A **mixed number** is a number consisting of a whole number and a fraction; $7\frac{1}{6}$, $9\frac{3}{4}$, $31\frac{5}{7}$.

A fraction is an expression of division. Thus **twelve divided by three** may be written $3 \overline{)12}$, or $12 \div 3$, or $\frac{12}{3}$.

Tell the proper fractions, the improper fractions, and the mixed numbers: $\frac{1}{2}$, $\frac{3}{3}$, $\frac{7}{8}$, $\frac{9}{3}$, $2\frac{1}{3}$, $\frac{8}{9}$, $6\frac{3}{4}$, $\frac{12}{4}$, $7\frac{1}{2}$, $\frac{5}{5}$, $\frac{16}{8}$, $\frac{3}{3}$, $\frac{20}{5}$, $\frac{6}{9}$, $20\frac{1}{6}$, $\frac{10}{2}$, $\frac{2}{1}$, $31\frac{1}{9}$, $\frac{9}{9}$, $1\frac{1}{3}$, $\frac{17}{18}$, $\frac{25}{5}$, $\frac{21}{3}$, $\frac{3}{20}$, $14\frac{8}{9}$, $\frac{30}{30}$, $5\frac{4}{5}$, $\frac{22}{11}$, $9\frac{4}{5}$, $\frac{1}{8}$, $\frac{16}{8}$, $\frac{20}{4}$.

PAPER

Write with a soft lead pencil or with pen and ink. If a pencil is so hard that its graphite must be softened by moisture from any source, it is too hard for the use to which it is being put.

Most children, if asked to try, soon make figures both clear and beautiful.

5 8 6 3 1 2 4 9 7 0

1. Write a *number table* from 1 to 100.
2. Write all the *odd* numbers in a *number table* from 1 to 99.
3. Write all the *even* numbers in a *number table* from 2 to 200.
4. Write *number tables*, counting: by threes to 300; by fours to 400; by fives to 500; by sixes to 600; by sevens to 700; by eights to 800; by nines to 900; by tens to 1000; by elevens to 1100; and by twelves to 1200.

SPELLING OF NUMBER NAMES

5. 1, one. 2, two. 3, three. 4, four. 5, five. 6, six. 7, seven. 8, eight. 9, nine. 10, ten. 0, zero. 11, eleven. 12, twelve. 13, thirteen. 14, fourteen. 15, fifteen. 16, sixteen. 17, seventeen. 18, eighteen. 19, nineteen. 20, twenty. 30, thirty. 40, forty. 50, fifty. 60, sixty. 70, seventy. 80, eighty. 90, ninety. 100, one hundred. 1000, one thousand. Spell orally as well as in writing.
6. Write in words all the numbers in questions 1, 2, 3, and 4 on page 10. Spell the words carefully.

NUMERATION

It is usual, in writing large numbers, either to point off the periods with commas, 428,532,829, or to leave a space between the periods, 428 532 829. This is simply as a convenience for the eye of the reader.

Only a few centuries ago, words even were not separated by spaces, and sentences were not begun with capitals or marked at the ends with periods.

1. Read 9 291 565 432 ; 5 827 002 472 ; 902 873 429 826.
2. The stocks and bonds of the United States Steel Corporation exceed \$1 400 000 000. Read these figures.
3. The average distance of the earth from the sun is 91 430 000 miles. Read these figures.
4. If the size of the United States is 2500 miles by 1300 miles, how many square miles does it contain? Since each square mile contains 640 acres, how many acres in all are there?
5. One man's fortune was $\frac{1}{7}$ that of another man. The second man was worth, in the money value of his property, \$700,000. What was the first man worth?
6. A farmer sold 8 cords of wood to one buyer, $2\frac{1}{2}$ times as much to another buyer, and $\frac{3}{4}$ as much to a third. How much wood did he sell to each of the second and third buyers?
7. A boy who had $\$2\frac{1}{2}$ paid $\$1\frac{3}{4}$ for three fine hens. How many dollars had he left? Was this more or less than one dollar? By how much?
8. What is ratio?
9. On an Italian coin are these letters, MDCCCLIV. In what year was it minted?

FRACTIONS



Point out halves and quarters.



Point out fifths and tenths.



Point out thirds and sixths.



Point out halves, quarters, and eighths.



Point out halves, thirds, fourths, and twelfths.



Point out sevenths.



Point out thirds and ninths.



Point out twentieths, tenths, and fifths.

EXERCISES

1. Draw a one-inch line. Add $\frac{1}{2}$ to it; $\frac{1}{4}$; $\frac{1}{8}$. How long is the whole line?
2. Draw a two-inch line. Add $\frac{1}{4}$ to it; $\frac{1}{8}$; $\frac{1}{16}$. How long is the whole line?
3. Draw a three-inch line. Add $\frac{1}{3}$ to it; $\frac{1}{6}$; $\frac{1}{9}$; $\frac{1}{18}$. How long is the whole line?
4. Draw a five-inch line. Add $\frac{1}{5}$ to it; $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{40}$. How long is the whole line?
5. Divide the five-inch line by 2. How long is each part?
6. Draw a six-inch line. Divide each inch into $\frac{1}{3}$ inches. How many $\frac{1}{3}$ inches are there in all?
7. Explain what you have done in each case.

COUNTING BY LARGE NUMBERS TO 200

13

1. 13, 26, 39, 52, 65, 78, 91, 104, 117, 130, 143, 156, 169, 182, 195.

2. How many 13's are there in each of these numbers?

14

3. 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196.

4. How many 14's are there in each of the numbers?

15

5. 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195.

6. How many 15's are there in each of the numbers?

16

7. 16, 32, 48, 64, 80, 96, 112, 128, 144, 160, 176, 192.

8. How many 16's are there in each of the numbers?

17

9. 17, 34, 51, 68, 85, 102, 119, 136, 153, 170, 187.

18

10. 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198.

19

11. 19, 38, 57, 76, 95, 114, 133, 152, 171, 190.

20

12. 20, 40, 60, 80, 100, 120, 140, 160, 180, 200.

24

13. 24, 48, 96, 120, 144, 168, 192.

25

14. 25, 50, 75, 100, 125, 150, 175, 200.

SUBTRACTION

Any number of numbers may be added at one time; but we can subtract, multiply, and divide only two numbers at any one time.

In subtracting, we call the number from which another number is taken the **minuend**, and the number taken away the **subtrahend**. The result we call the **difference**, or *remainder*.

PROOFS

The *difference* and the *subtrahend* together always equal the *minuend*. $18 - 6 = 12$, $12 + 6 = 18$.

The *minuend* less the *difference* always equals the *subtrahend*. $18 - 6 = 12$, $18 - 12 = 6$.

QUESTIONS

WRITE

Prove all answers in two ways.

- | | |
|------------------------|------------------------------|
| 1. $1857 - 289 = ?$ | 4. $\$349.27 - \$287.36 = ?$ |
| 2. $10940 - 1111$. | 5. $\$800.00 - \$732.80$. |
| 3. $40,000 - 28,931$. | 6. $\$432. - \$381.25$. |

7. Mr. William Thompson, when he was twenty-one years of age, inherited \$1,297,856, which was given into his possession by the trustees of his father's property. He found that \$829,642 was invested in land and buildings. How much was invested in other property, such as railroad shares and government bonds?

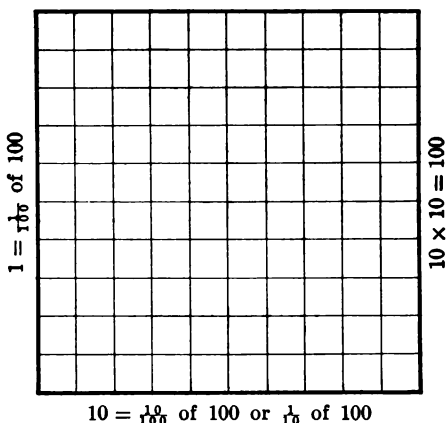
8. A great city owed \$10,675,431, and the property of all its people was considered worth \$231,482,649. What was the difference between all the private property and the public debts?

HUNDRED AND HUNDREDTHS

We saw that $\frac{1}{2}$ of 2 things is 1, that $\frac{1}{5}$ of 5 things is 1, and that $\frac{1}{10}$ of 10 things is 1.

Every whole number suggests a fraction like it in name.

The number one hundred suggests a hundredth as a fraction.



1. Point out $\frac{1}{100}$, $\frac{1}{10}$, $\frac{3}{100}$, $\frac{3}{10}$, $\frac{25}{100}$, $\frac{33}{100}$, $\frac{50}{100}$.
2. Draw three squares like this and divide each of them into 100 squares.
 - a. Mark on one of them one half of the squares blue or red or black. How many are one half of one hundred?
 - b. On another square mark one fourth blue, another fourth red, and another fourth black. How many are one fourth of one hundred?
 - c. On the last square mark one third blue and another third red. How many are left white? If $3 \times 33 = 99$, then $\frac{1}{3}$ of 100 = ? Mark the last hundredth into thirds. What does this show? $33\frac{1}{3} \times 3 = ?$

PER CENTS

The fraction, a hundredth, is so important that we have another name for it, a **per cent**. This means *by the hundred*. 5 per cent is $\frac{5}{100}$. 10 per cent is $\frac{10}{100}$. $33\frac{1}{3}$ per cent is thirty-three and one third hundredths. Just as we have ¢ as the sign for cent and \$ as the sign for dollar or 100¢, so also we have a sign for hundredths or per cents. This sign for hundredths is %, called **per cent**.

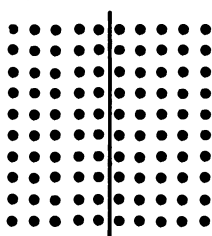
6% of \$1 is $\frac{6}{100}$ of 100¢, or 6¢

50% of \$1 is $\frac{50}{100}$ of 100¢, or 50¢

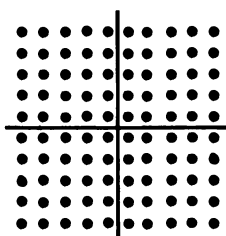
Per cents are especially used in dealing with money; for when one lends money to another, the lender usually asks the borrower not only to give him back after a time all his money, but also to pay him so many per cent for the use of the money. This payment is called **interest**. Also we pay the governments of our town or city and of our State every year so many per cent of the money value of our property. This payment is called a **tax**. It supports the police and schools and takes care of the streets or roads.

4% of \$1000 is $\frac{4}{100}$ of \$1000 $\frac{4 \times 1000}{100} = \$40$ $\$10 \times 4 = \$40$

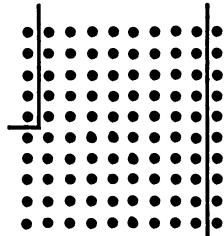
2% of \$5000 is $\frac{2}{100}$ of \$5000 $\frac{2 \times 5000}{100} = \$100$ $\$50 \times 2 = \$100$



50%



25%



5%

10%

ALIQOT PARTS OF 100

1. $\frac{100}{3} = 33\frac{1}{3}$ $33\frac{1}{3}$, $66\frac{2}{3}$, 100 $33\frac{1}{3} \times 3 = 100$.
2. $\frac{100}{6} = 16\frac{2}{3}$ $16\frac{2}{3}$, $33\frac{1}{3}$, 50, $66\frac{2}{3}$, $83\frac{1}{3}$, 100.
3. $\frac{100}{8} = 12\frac{1}{2}$ $12\frac{1}{2}$, 25, $37\frac{1}{2}$, 50, $62\frac{1}{2}$, 75, $87\frac{1}{2}$, 100.
4. $\frac{100}{12} = 8\frac{1}{3}$ $8\frac{1}{3}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, $41\frac{2}{3}$, 50.
5. $\frac{100}{16} = 6\frac{1}{4}$ $6\frac{1}{4}$, $12\frac{1}{2}$, $18\frac{3}{4}$, 25, $31\frac{1}{4}$, $37\frac{1}{2}$, $43\frac{3}{4}$, 50.
6. $16\frac{2}{3} \times 6 = ?$ $12\frac{1}{2} \times 8 = ?$ $8\frac{1}{3} \times 12 = ?$ $6\frac{1}{4} \times 16 = ?$
7. What is the ratio of $\frac{1}{6}$ of 100 to $\frac{1}{12}$ of 100?
8. What is the ratio of $\frac{1}{16}$ of 100 to $\frac{1}{8}$ of 100?
9. What part of 100 is $37\frac{1}{2}$? $87\frac{1}{2}$? 75?
10. What part of 100 is $16\frac{2}{3}$? $41\frac{2}{3}$? $83\frac{1}{3}$?
11. What part of 100 is $6\frac{1}{4}$? $31\frac{1}{4}$? $37\frac{1}{2}$?

12. John bought 100 marbles. He gave $\frac{1}{6}$ to his brother Charles, $\frac{1}{10}$ to his brother Will, and $\frac{1}{4}$ to his brother Thomas. How many had he left?

13. Mary had one dollar and bought two yards of wide silk ribbon at $37\frac{1}{2}$ ¢ per yard. What part of a dollar should she receive in change?

REVIEW

WRITE

14. Find the population of a town which has 16,009 girls, 15,700 boys, 35,019 women, and 37,009 men.

15. In 6729 eggs are how many dozen?

16. Seven thousand bushels of wheat, 16,009 of barley, nine thousand twelve of oats, 6019 of beans, 14,076 of peas, 8019 of rye, and 756 of buckwheat, are how many bushels in all?

PERCENTAGE

How many cents are there in a dollar? What part of a dollar is one cent?

3¢ are how many hundredths of a dollar?

6¢ are how many hundredths of a dollar?

7 hundredths of a dollar are how many cents?

1 hundredth of a dollar is one per cent of it.

2 hundredths of any number is two per cent of it.

4 hundredths of any number is four per cent of it.

27 per cent of a number is 27 hundredths of it.

Any per cent of a number is an equal number of hundredths of that number.

In business the sign % is used for *per cent*.

Per cent or % may be read *hundredths*.

3 per cent = 3%; 6 per cent = 6%;

10 per cent = 10%; 45 per cent = 45%;

91 per cent = 91%; 100 per cent = 100%.

Any rate per cent, being a rate per hundred, may be written as a common fraction or a decimal fraction.

1 per cent = $\frac{1}{100}$ = 1% = .01.

7 per cent = $\frac{7}{100}$ = 7% = .07.

34 per cent = $\frac{34}{100}$ = 34% = .34.

What is $16\frac{2}{3}\%$ of 96 bu.? $16\frac{2}{3} \times 6 = 100$.

$16\frac{2}{3}\%$ of any number is $\frac{1}{6}$ of the number.

$6 \overline{)96}$
16

Dividing 96 by 6 we obtain 16 bu., the required percentage of 96 bu.

MULTIPLICATION TABLES

TWOS	THREES	FOURS	FIVES
$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$	$1 \times 5 = 5$
$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$
$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$	$3 \times 5 = 15$
$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 5 = 20$
$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$	$5 \times 5 = 25$
$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$	$6 \times 5 = 30$
$7 \times 2 = 14$	$7 \times 3 = 21$	$7 \times 4 = 28$	$7 \times 5 = 35$
$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$	$8 \times 5 = 40$
$9 \times 2 = 18$	$9 \times 3 = 27$	$9 \times 4 = 36$	$9 \times 5 = 45$
$10 \times 2 = 20$	$10 \times 3 = 30$	$10 \times 4 = 40$	$10 \times 5 = 50$
$11 \times 2 = 22$	$11 \times 3 = 33$	$11 \times 4 = 44$	$11 \times 5 = 55$
$12 \times 2 = 24$	$12 \times 3 = 36$	$12 \times 4 = 48$	$12 \times 5 = 60$
$13 \times 2 = 26$	$13 \times 3 = 39$	$13 \times 4 = 52$	$13 \times 5 = 65$
$14 \times 2 = 28$	$14 \times 3 = 42$	$14 \times 4 = 56$	$14 \times 5 = 70$
$15 \times 2 = 30$	$15 \times 3 = 45$	$15 \times 4 = 60$	$15 \times 5 = 75$
$16 \times 2 = 32$	$16 \times 3 = 48$	$16 \times 4 = 64$	$16 \times 5 = 80$
$17 \times 2 = 34$	$17 \times 3 = 51$	$17 \times 4 = 68$	$17 \times 5 = 85$
$18 \times 2 = 36$	$18 \times 3 = 54$	$18 \times 4 = 72$	$18 \times 5 = 90$
$19 \times 2 = 38$	$19 \times 3 = 57$	$19 \times 4 = 76$	$19 \times 5 = 95$
$20 \times 2 = 40$	$20 \times 3 = 60$	$20 \times 4 = 80$	$20 \times 5 = 100$

It is often convenient to know the tables from 13 to 20.

MULTIPLICATION TABLES

SIXES	SEVENS	EIGHTS	NINES
$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$	$1 \times 9 = 9$
$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$	$2 \times 9 = 18$
$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$	$3 \times 9 = 27$
$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$
$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$	$5 \times 9 = 45$
$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$	$6 \times 9 = 54$
$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$	$7 \times 9 = 63$
$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$
$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$	$9 \times 9 = 81$
$10 \times 6 = 60$	$10 \times 7 = 70$	$10 \times 8 = 80$	$10 \times 9 = 90$
$11 \times 6 = 66$	$11 \times 7 = 77$	$11 \times 8 = 88$	$11 \times 9 = 99$
$12 \times 6 = 72$	$12 \times 7 = 84$	$12 \times 8 = 96$	$12 \times 9 = 108$
$13 \times 6 = 78$	$13 \times 7 = 91$	$13 \times 8 = 104$	$13 \times 9 = 117$
$14 \times 6 = 84$	$14 \times 7 = 98$	$14 \times 8 = 112$	$14 \times 9 = 126$
$15 \times 6 = 90$	$15 \times 7 = 105$	$15 \times 8 = 120$	$15 \times 9 = 135$
$16 \times 6 = 96$	$16 \times 7 = 112$	$16 \times 8 = 128$	$16 \times 9 = 144$
$17 \times 6 = 102$	$17 \times 7 = 119$	$17 \times 8 = 136$	$17 \times 9 = 153$
$18 \times 6 = 108$	$18 \times 7 = 126$	$18 \times 8 = 144$	$18 \times 9 = 162$
$19 \times 6 = 114$	$19 \times 7 = 133$	$19 \times 8 = 152$	$19 \times 9 = 171$
$20 \times 6 = 120$	$20 \times 7 = 140$	$20 \times 8 = 160$	$20 \times 9 = 180$

Write these tables from memory.

REVIEW

1. Count by 18 to 200.
2. What is $\frac{1}{6}$ of 24, 36, 72, 30, 96, 48, 60, 84?
3. Read 32,498,721.
4. Tell the figures for three billion nine hundred million.
5. The sun is over 800,000 miles in diameter. How many miles is it in circumference? $3\frac{1}{7} \times 800,000 = ?$
6. The earth is 25,000 miles in circumference. About how many miles is it in diameter? $25,000 \div 3\frac{1}{7} = ?$
7. If the United States is 2500 miles wide from New York to San Francisco, what is the ratio of its breadth to the circumference of the earth?
8. Since the earth is 8000 miles in diameter, what is the ratio of its diameter to that of the sun?
9. If a good silver watch costs \$8, and a good gold watch \$40, how many silver watches can be bought for the price of one gold watch?
10. If the United States has about seventy-five million people, and if China has at least three hundred million, what is the ratio of our population to that of China?
11. If one book cost five dollars, and a set of sixteen books, of a different kind, cost four dollars, what is the ratio of the cost of the first book to that of one of the other books?
12. Mr. Williams bought 100 M. bricks for \$800, and sold 60 M. for \$540. Did he make or lose per M.?
13. A contractor employed a carpenter at \$3 per day, and charged \$3.50 in the bill for his work. What was the ratio of the contractor's gain to the workman's wages? to the charge in the bill?

QUESTIONS

1. If one family burns one ton of coal a month and another family burns two tons in half a year, what is the ratio of the coal used by the first family to that used by the second?

2. What is the yearly cost of coal for a family that uses on the average $1\frac{1}{2}$ T. per mo. @ \$5 per T? What is the cost to another family using $\frac{2}{3}$ as much?

3. If a barrel of flour weighs 196 pounds, what is the weight of a bag of flour containing one eighth of a barrel?

4. If flour is \$4.80 per bbl., what is the cost of $\frac{1}{8}$ bbl.?

5. In a school of 600 children there was an average of 30 children in each room. How many rooms were there?

6. A boy's father bought for him a bicycle costing \$25. At the same price, what would the father have to pay for four bicycles for four sons?

7. Which is lower in price per sheep: 40 head of sheep for \$320, or 12 head for \$144?

8. A man who wished to build a house was offered two different lots of land: one was 60 feet wide, price \$900, the other was 40 feet wide, price \$800. Which was cheaper per front foot? Make drawing on blackboard.

9. A little boy paid a dime for a dozen marbles, though another boy offered him seven marbles for a nickel. Which marbles were cheaper?

10. One room has 15 square yards of carpet, another has 16 square yards. What may be the dimensions of each room in feet? Make drawings on blackboard.

QUESTIONS

1. Beginning at 5, count by 15 to 200.
2. Give the Multiplication Table of Fours.
3. A milkman sold 30 pts. of cream at the rate of 40¢ a quart in cash. How much money did he receive?
4. In a certain city there are 12 blocks of buildings to a mile. What is the ratio of 4 of these blocks to a mile of them?
5. In which city are the blocks longer, one which has 16 blocks, or one which has 8 blocks, to the mile?
6. A city block is 400 feet long, and each house lot in it is supposed to be 20 feet wide. How many house lots has such a block?
7. In a certain village it is $\frac{1}{4}$ of a mile from the church to the post office, $\frac{1}{8}$ of a mile from the school to the church, and $\frac{1}{2}$ of a mile from the post office to the school. Which is the longest, and which the shortest distance?
8. If this line ————— represents a mile, how many miles is it from A to B?

A scale is a ratio.

9. Upon a scale of one quarter inch to a foot, how many feet would a real foot represent?
10. Upon the same scale, how many feet would 5 inches represent?

When men plan houses and buildings, they make drawings to scale, sometimes $\frac{1}{4}$ in. to 1 ft., sometimes $\frac{1}{8}$ in., or $\frac{1}{16}$ in., or even $\frac{1}{32}$ in. to 1 ft.

11. If a house is to be 40 feet wide, by which scale would the plans be largest? By which smallest? Why?

MULTIPLICATION TABLES

TENS

To multiply by ten, add a zero at the right of the figures which represent the number. This advances each figure to an order one place higher. Each place shows a multiple by ten of the next lower place.

ELEVENS

To multiply by eleven, in the case of a number represented by a single figure, repeat the figure twice.	$11 \times 11 = 121$	$16 \times 11 = 176$
	$12 \times 11 = 132$	$17 \times 11 = 187$
	$13 \times 11 = 143$	$18 \times 11 = 198$
	$14 \times 11 = 154$	$19 \times 11 = 209$
	$15 \times 11 = 165$	$20 \times 11 = 220$

TWELVES

$1 \times 12 = 12$	$6 \times 12 = 72$	$11 \times 12 = 132$	$16 \times 12 = 192$
$2 \times 12 = 24$	$7 \times 12 = 84$	$12 \times 12 = 144$	$17 \times 12 = 204$
$3 \times 12 = 36$	$8 \times 12 = 96$	$13 \times 12 = 156$	$18 \times 12 = 216$
$4 \times 12 = 48$	$9 \times 12 = 108$	$14 \times 12 = 168$	$19 \times 12 = 228$
$5 \times 12 = 60$	$10 \times 12 = 120$	$15 \times 12 = 180$	$20 \times 12 = 240$

QUESTIONS

1. $12 \times 11 = ?$ 2. $16 \times 11 = ?$ 3. $19 \times 11 = ?$

Notice that any multiple by 11, of a number from 11 to 18, is represented by three figures, of which the hundreds' figure is that of the tens of the multiplicand, the tens' figure represents the sum of the two figures of the multiplicand, and the units' figure is the units' figure of the multiplicand.

Sometimes noticing these curiosities of arithmetic makes the study more interesting.

GENERAL MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8								
3	6	9	12								
4	8	12	16								
5					30			45			
6				30							72
7							56				
8						56	64				
9			36						90		
10								90			
11										121	
12					72						144

1. On a sheet of paper mark off 144 half-inch squares. Copy very accurately the numbers here. Fill in each blank square by the multiple of the numbers at the head of the column and at the left end of the row. $4 \times 2 = 8$ and $2 \times 4 = 8$. $11 \times 11 = 121$ and $12 \times 12 = 144$.

2. Compare your results with the multiplication tables in this book.

3. On the blackboard make 144 two-inch squares and proceed as in 1.

4. Why are the numbers larger, the nearer they are to the lower right hand corner of the *table*?

MULTIPLYING BY 10; 100; 1000

1. $562 \times 10 = ?$

$$\begin{array}{r} 562 \\ 10 \\ \hline 5620 \end{array}$$

2. $3894 \times 10 = ?$

$$\begin{array}{r} 3894 \\ 10 \\ \hline 38940 \end{array}$$

To multiply by 10, add a zero at the right.

3. $8729 \times 10 = ?$

4. $45,287 \times 10 = ?$

5. $900 \times 10 = ?$

6. $\$562.25 \times 10 = ?$

$$\begin{array}{r} \$562.25 \\ 10 \\ \hline \$5622.50 \end{array}$$

7. $\$3894.40 \times 10 = ?$

$$\begin{array}{r} \$3894.40 \\ 10 \\ \hline \$38944.00 \end{array}$$

To multiply by 10 a sum of money expressed in dollars and cents, add a zero at the right and move the decimal point one place to the right.

8. $\$432.50 \times 10 = ?$

9. $\$92.75 \times 10 = ?$

10. $349 \times 100 = ?$

$$\begin{array}{r} 349 \\ 100 \\ \hline 34900 \end{array}$$

11. $4876 \times 1000 = ?$

$$\begin{array}{r} 4876 \\ 1000 \\ \hline 4876000 \end{array}$$

To multiply by 100, add two zeros at the right.

To multiply by 10, 100, 1000, 10,000, 100,000, 1,000,000, etc., add as many zeros at the right as the multiplier contains zeros.

12. Multiply 472, 8349, 5461, 289, and 4567 by 100, 1000, 10,000 and 100,000.

To multiply by 10, 100, 1000, 10,000, 100,000, 1,000,000, sums of money expressed in dollars and cents, move the decimal point in the product as many places farther to the right as there are zeros in the multiplier.

QUESTIONS

1. Multiply \$329 by 1000, and divide the product by 7.
2. Write the Multiplication Table of Elevens.
3. What is $\frac{1}{8}$ of 64, 96, 80, 32, 24, 48, 88, 72, 56?
4. What is the ratio to 12 of 108, 84, 144, 60?
5. Draw a plan for a room 16 ft. $\times$ 12 ft. to the scale of $\frac{1}{4}$ in. to 1 ft.
6. A man's property at his death was found to be worth \$90,000. He left a wife and five children. To the widow he gave $\frac{1}{3}$ of the property, and to each of his children $\frac{1}{5}$ of the remainder. How much did each receive?
7. Which is greater, one hundred forty million or one thousand times ten thousand? What is the ratio of the first number to the product of the other numbers?
8. If one thousand men earn by work \$2 a day each, and another man effects a saving in a great business so that he receives for his work \$2000 a week, what is the ratio of the weekly earnings of all the thousand men for six days to the salary of the one man?
9. Is it more to save \$10 a month or to save \$3 a week? Why?
10. There are 800 pages in one book which costs \$4, and 300 pages in another book which costs \$2. If the pages of each book are equally valuable, which book is cheaper?
11. One man bought 2 doz. fresh eggs for 40¢, another man bought $\frac{1}{2}$ doz. for 12¢. Which eggs were cheaper?
12. A boy wore out 4 pr. of \$2 shoes a year, another boy wore out 6 pr. of \$1.50 shoes, and still another boy wore out 3 pr. of \$3 shoes. Which boy's shoes cost his father the most money a year, if the cost of repairs is not counted?

MULTIPLICATION

1. Multiply 15,468 by 627.

$$\begin{array}{rcll}
 15468 & & \text{multiplicand} \\
 627 & & \text{multiplier} \\
 \hline
 7 \times 15468 = 108276 & & \text{first partial product} \\
 20 \times 15468 = 30936 & & \text{second partial product} \\
 600 \times 15468 = 92808 & & \text{third partial product} \\
 \hline
 627 \times 15468 = 9698436 & & \text{total product}
 \end{array}$$

2. Multiply 29,546 by 703.

$$\begin{array}{rcll}
 29546 & & \text{multiplicand} \\
 703 & & \text{multiplier} \\
 \hline
 3 \times 29546 = 88638 & & \text{first partial product} \\
 700 \times 29546 = 206822 & & \text{third partial product} \\
 \hline
 703 \times 29546 = 20770838 & & \text{total product}
 \end{array}$$

$$\begin{array}{r}
 3. \quad 46227 \\
 \quad 8006 \\
 \hline
 \quad 277362 \\
 369816 \\
 \hline
 370093362
 \end{array}$$

$$\begin{array}{r}
 4. \quad 88743 \\
 \quad 5600 \\
 \hline
 \quad 53245800 \\
 443715 \\
 \hline
 496960800
 \end{array}$$

Since we cannot multiply by zeros, which indicate absence of value in the places they occupy, we set down the zeros to show where the next

product of the multiplication is to begin.

5. Mary's father bought 640 acres of land for \$175 an acre. What was the cost of the section of land?

6. The government employed 34,529 private soldiers at a cost in board and wages of \$31.90 a month each on the average. In one year what was the total cost of the private soldiers for these two expenses?

7. Multiply 43,287 by 192, 147, 298, 3753, and 4665.

MULTIPLICATION

Multiplication repeats one number as many times as there are units in another.

The number repeated, or multiplied, is the **multiplicand**.

The number showing how many times the multiplicand is repeated is the **multiplier**.

The result of the multiplication is the **product**.

The sign of multiplication, $\times$, is read **multiplied by**. It is also read *times*.

$75 \times 5 = ?$ may be read: Five times seventy-five, or Seventy-five multiplied by five, is what?

$1896 \times 245 = ?$ We may say either: Find 245 times 1896; or, Multiply 1896 by 245. It is usually better to read, Multiply.

Multiply:

1. 7019 by 203, 405, 607, 809, 506, 705.
2. 45,608 by 314, 415, 516, 617, 819, 218.
3. 45,609 by 340, 450, 560, 670, 780, 890.
4. 317,018 by 234, 345, 456, 567, 789, 895.
5. 49,017 by 451, 671, 891, 341, 751, 861.
6. 68,907 by 237, 459, 738, 426, 954, 765.

7. There were one hundred nine schools in the city of Brighton, and the average number of pupils in each school was two hundred ninety-eight. How many pupils were there in all?

8. There were nine hundred forty-six houses in the town of Walton, and the average number of panes of glass in each house was one hundred eighty-three. How many panes of glass were there in all?

9. If in each house two panes of glass were broken in each year, how many new panes in all were needed?

MULTIPLICATION

When we have multiplied two numbers together, one way to find whether our product is probably correct is to use the multiplier as multiplicand, and to multiply the numbers again.

652	Multiplicand	Proof:	314
314	Multiplier		652
2608			628
652			1570
1956			1884
204728	Product		204728

Multiply and prove in this way:

- | | | | | | |
|-----------|-----------|------------|------------|-------------|------------|
| 1. 48 | 2. 97 | 3. 149 | 4. 556 | 5. 1242 | 6. 2987 |
| <u>36</u> | <u>15</u> | <u>167</u> | <u>497</u> | <u>1298</u> | <u>260</u> |

7. Multiply 3291 by 8272. Prove by exchanging multiplier and multiplicand.

8. Multiply 4875 by 9291; and prove the answer.

9. In a great battle there were engaged on both sides together two hundred sixteen regiments with an average of six hundred fifty soldiers in each regiment. How many soldiers were engaged in all?

10. If one fifth of all the soldiers engaged were killed or wounded or deserted to go home, how many were left in both armies?

11. If in 1900 there were about 84,000,000 in all the United States and its possessions, and if 16,000,000 of these were colored people, what was the ratio of the white people to the colored people?

ROMAN NOTATION

This notation was first used by the ancient Romans. It employs seven capital letters.

In the prefaces of books small letters are used to mark the pages.

Our more common figures came from Arabia and India, lands even farther away than Rome in Italy.

$I=1$; $V=5$; $X=10$; $L=50$; $C=100$; $D=500$; $M=1000$.

TABLE

$I=1$	$XX=20$	$LXXI=71$	Compare these two ways of writing numbers and give some reasons why the Roman notation is easier and some reasons why it is harder than our common no- tation.
$IV=4$	$XXI=21$	$LXXXI=81$	
$V=5$	$XXX=30$	$XC=90$	
$VI=6$	$XXXI=31$	$XCI=91$	
$IX=9$	$XL=40$	$CI=101$	
$X=10$	$XLI=41$	$CC=200$	
$XI=11$	$L=50$	$D=500$	
$XIV=14$	$LI=51$	$DC=600$	
$XV=15$	$LX=60$	$CM=900$	
$XVI=16$	$LXI=61$	$M=1000$	
$XIX=19$	$LXX=70$	$MM=2000$	

PRINCIPLES

Repeating a letter repeats its value :

$III, 3$; $XXX, 30$; $CC, 200$; $CCCXXII, 322$.

When a letter is placed before one of greater value, the value of the less number is subtracted from the value of the greater ; as, $IV=5-1=4$; $XL=50-10=40$.

$XIX=10+10-1=19$; $XXIV=10+10+5-1=24$.

When a letter is placed after one of greater value, the value of the less is added to the value of the greater ; as,

$VI=5+1=6$; $MC=1000+100=1100$.

LONG MEASURE

Long measure is sometimes called **linear measure**. It is used in measuring length or distance.

$$12 \text{ inches (in.)} = 1 \text{ foot (ft.)}$$

$$3 \text{ feet} = 1 \text{ yard (yd.)}$$

$$5\frac{1}{2} \text{ yards or } 16\frac{1}{2} \text{ feet} = 1 \text{ rod (rd.)}$$

$$320 \text{ rods or } 5280 \text{ feet} = 1 \text{ mile (mi.)}$$

1. How many inches are there in 2 ft.? 3 ft.? 7 ft.?
2. How many yards are there in 2 rd.? 6 rd..? 5 rd.?
3. How many rods are there in 2 miles? 3 miles?
8 $\frac{1}{2}$ miles? 100 miles?
4. How many inches are there in 1 yd.? 2 yd.?
1 $\frac{1}{2}$ yd.? 2 $\frac{1}{2}$ ft.? 2 $\frac{1}{4}$ yd.?
5. How many feet are there in 2 rd.? 4 rd.?
6. How many feet are there in 18 mi. 19 rd.?

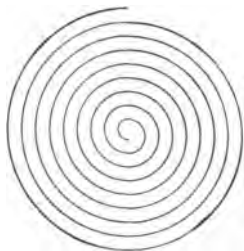
$$\begin{array}{r}
 18 \text{ mi.} \\
 \underline{320} \\
 360 \\
 54 \\
 \underline{5760 \text{ rd.}} \\
 19 \text{ rd.} \\
 \underline{5779 \text{ rd.}} \\
 16\frac{1}{2} \\
 \underline{2889\frac{1}{2}} \\
 34674 \\
 5779 \\
 \underline{95353\frac{1}{2} \text{ ft.}}
 \end{array}$$

As there are 320 rd. in a mile, in 18 miles there are 320 times 18 rd., or 5760 rd. Adding the 19 rd., we obtain 5779 rd. As there are 16 $\frac{1}{2}$ ft. in 1 rd., in 5779 rd. there are 16 $\frac{1}{2}$ times 5779 ft., or 95,353 $\frac{1}{2}$ ft. Hence in 18 miles 19 rd. there are 95,353 ft.

In multiplying by 16 $\frac{1}{2}$, it is convenient to take $\frac{1}{2}$ of the multiplicand first.
$$\begin{array}{r}
 2)5779 \\
 \underline{2889\frac{1}{2}}
 \end{array}$$

To this we add the product of 5779 by the whole number, 16.

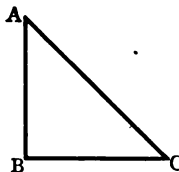
LENGTHS



Not all things to be measured are in straight lines.

A clock spring, 18 inches long, would be coiled up like this. Coil up 3 yards of thread or string, and see how it looks.

RIGHT ANGLE TRIANGLES



If we wish to cross a street and to save time and distance, we cut corners. If we wish to go to C from A , we may go from A to B and from B to C . Make drawings of triangles and measure.

The lines AB , BC , and CA form the **perimeter** of the triangle ABC .

BC is the **base**. AB is the **height**.

AC is the **hypotenuse**. The angle ABC is a **right angle**.

The angles CAB and ACB are **acute angles**.

The triangle ABC is called **equilateral** (equal sided) because it has two equal sides.

1. Draw a triangle: AB 3 in., BC 4 in. Measure AC .

2. $AB = 4$ in. $BC = 5$ in. $AC = ?$

3. $AB = 3$ miles. $BC = 6$ miles. $AC = ?$

Draw this by a scale 1 inch to 1 mile.

4. Draw another right angle triangle, AB 2 in., BC 8 inches. $AC = ?$ Upon each side of the triangle make a square divided into sq. in. Count all the sq. in.

QUESTIONS

1. How many times longer is a mile than a yard?
2. What is the ratio of a mile to a rod?
3. What is the ratio of a mile to 16 rods?
4. What is the ratio of a yard to an inch?
5. A boy's garden was 33 ft. wide by 66 ft. long. Give its dimensions in rods. Draw to scale $\frac{1}{4}$ in. = 1 ft.
6. One boy took a running jump and cleared 3 yd. Another jumped 100 inches. Which jumped farther? By how much?
7. A man walked 12 miles in 3 hr., and a second man walked 27 miles in $\frac{3}{8}$ of a day. Which one walked at the faster rate? What was the number of miles each walked in one hour?
8. What is the ratio of A's speed of 5 miles an hour and B's speed of 6 miles an hour in running? Be sure that your fraction which tells the ratio shows clearly which man runs the faster.

REVIEW

WRITE

9. Review some of the additions on page 18.
10. Count by 24 to 200.
11. Write in words 251,863,007,521 and 70,001,989,502.
12. Subtract \$5861.90 from \$6000; \$199,854 from \$250,000; and \$367.15 from \$10,300.
13. To the scale of 1 in. to 1 ft. draw a square $5\frac{1}{2}$ ft. by 6 ft. Divide it by the scale into $\frac{1}{2}$ ft. squares.
14. Multiply 8432 by 2379 and prove by exchanging multiplier and multiplicand and multiplying again.
15. Divide 863,400 by 1000.
16. Prove by drawings that $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$.

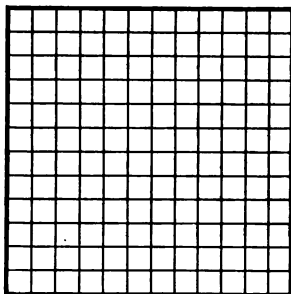
SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ sq. ft.	= 1 square rod (sq. rd.)
160 square rods, or 43,560 sq. ft.	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Square measure is used in measuring surfaces. A surface has only length and breadth; as, a sheet of paper, or one of the faces of a brick, or the face of a blackboard.

A square is a figure having four equal sides and four equal angles.

A square inch is a square each of whose sides is an inch long. A square foot is a square having each side 1 foot, or 12 inches long. Its contents are $12 \times 12 = 144$ square inches, that is, the square can be divided into 144 squares, each of whose sides is an inch long.



Scale $\frac{1}{12}$ in. to 1 in.

If the oblong or rectangle has sides 12 inches long and 2 inches wide, its square contents are equal to 12×2 inches, or 24 inches, that is, the rectangle is divisible into twenty-four squares, each of whose sides is an inch long. Hence, if the length and breadth of a square or of any other rectangle are given, the **area**, or contents, of the figure can be found by multiplying the length by the breadth.



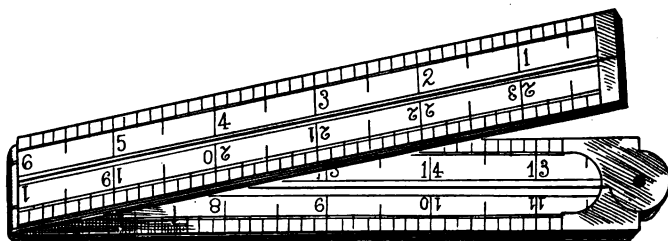
Scale $\frac{1}{12}$ in. to 1 in.

Land, plastering, paving, painting, carpeting, and cloth are measured by square measure.

QUESTIONS

1. What is the area of a plot of ground 30 ft. by 180 ft.?
2. Of a plot 45 ft. by 120 ft.?
3. Of a plot 60 ft. by 100 ft.?
4. A man who wished to get for his children a plot of ground with as many square feet in it as possible, was offered three plots all at the same price. One was 90 ft. wide, 120 ft. deep; another was 50 ft. wide, 200 ft. deep; the third was 40 ft. wide, 300 ft. deep. Which was the best for him to buy?
5. Mr. Thompson bought a great farm containing 50 sq. m. How many acres was that?
6. A square 2 feet on a side contains 576 sq. in. Show by a drawing to the scale of $\frac{1}{8}$ in. to 1 ft. why this is true. $\frac{1}{8}$ in. = how many inches?
7. Which is the larger area, a 5 in. sq. or 20 sq. in.? By how much?
8. A carpet contained 28 sq. yd. If one side of the room was 12 ft. long, what was the length of the other side?
9. If a section of land contains 640 acres, how many acres does a quarter section contain? a half section?
10. What is the length of the perimeter of a square mile? of $\frac{1}{4}$ sq. in.?
11. A farmer sold 80 acres of land for \$8000, but refused \$10,000 for $\frac{1}{8}$ of a section. Which was the higher price? By what ratio was it higher?
12. A contractor offered to make a stone road bed at \$30 a square rod. A competitor offered to make it at \$1 a square yard. Which price was lower?
13. At the first price what would have been the cost of 1000 sq. rd.? What at the second price?
14. Write the aliquot parts of 100 divided in eighths.

MEASURING



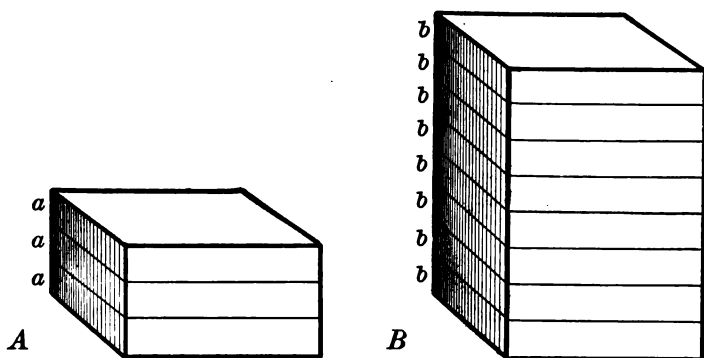
Folding Foot Rule

When we measure anything by long measure or square measure or cubic measure, we find its *ratio* to a fixed length, one inch, one foot, one yard, etc. Carpenters use “foot rules” and “2-foot rules.”

When we say that a piazza on a house is twenty feet long, we mean that its ratio to one foot is twenty.

1. Draw a line on the blackboard which you think is 1 ft. long. Measure it.
2. Cut a strip of paper 1 ft. long by 1 in. wide. Mark on it inches divided into sixteenths.
3. Make estimates of windows, doors, floors, in feet and inches. Measure them, and compare the results.
4. Measure the width of sidewalks and streets, and bring the results to school.
5. Measure the sizes of buildings.
6. Make to scale outline drawings of floors, desks, and dictionaries.
7. Measure the length of the track of a wheel making one revolution.

PRISMS



The *volume* or contents of a square *prism* are found by multiplying together the numbers which express its dimensions. A square prism is a cube all of whose faces are rectangles.

1. What is the ratio of A to B ? of B to A ?
2. If A is 100, what is a ?
3. If B is 100, what is b ?

The ratio of equals is one.

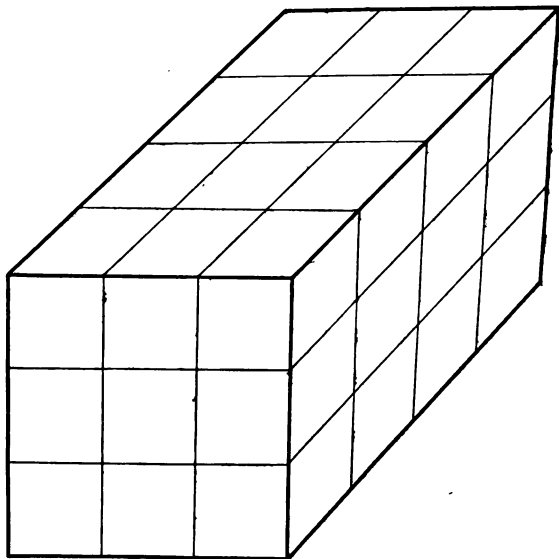
4. What is the ratio of a to b ? of b to a ?
5. If a is $33\frac{1}{3}\%$ of A , what is $a + a$?
6. If b is $12\frac{1}{2}\%$ of B , what is $b + b + b$?
7. Point out $87\frac{1}{2}\%$ of B .
8. Point out upon B 200% of A .
9. How would you draw $16\frac{2}{3}\%$ of A ? What part of A would that be?
10. Draw prisms of various ratios to each other.

CUBIC MEASURE

A **cube** has **length**, **breadth**, and **thickness**; six sides; and twelve edges, forming right angles with each other.

The sides are called **faces**.

The **volume** of a cube is found by multiplying together the numbers expressing its length, breadth, and thickness. The measures of the length, breadth, and thickness are called **dimensions**. The product is called **cubic inches**, **feet**, **yards**, etc.



Scale: $\frac{1}{3}$ in. = 1 ft.

The dimensions of this cube are, if we follow the scale:

length . . . 4 inches

breadth . . . 3 inches

thickness . . . 3 inches

Its volume is . . . 36 cubic inches

CUBIC MEASURE

Since a foot contains twelve inches, a cubic foot contains $12 \times 12 \times 12$ cubic inches. A cubic foot measures twelve inches in each dimension. $12 \times 12 \times 12 = 1728$.

Since a yard contains three feet, a cubic yard contains nine cubic feet.

TABLE

1728 cubic inches = 1 cubic foot (cu. ft.)

27 cubic feet = 1 cubic yard (cu. yd.)

128 cubic feet = 1 cord (cd.), used in
measuring wood

A **square prism** is a solid whose volume may be found by cubic measure. A **cord** of wood is piled in the form of a square prism 4 ft. by 4 ft. by 8 ft.

1. How many cubic inches are there in one cubic yard?
2. Find the number of cubic feet of space in a house 30 ft. wide, 40 ft. deep, and 22 ft. high.
3. A pile of wood measures 80 ft. by 40 ft. by 10 ft. How many cubic feet are there in its volume?
4. A fireplace holds 4 cu. ft. of wood. If this amount lasts an hour, how many hours does a cord of wood last?
5. In a coal mine a coal vein is 2 yd. thick, 5 yd. high, and 200 yd. long. How many cu. yd. are in the vein?
6. How many cu. in. are there in 100 cu. ft.?
7. How many cu. ft. of air are in a room 8 ft. $\times$ 8 ft. $\times$ 10 ft. in size?
8. How many cu. ft. are in a piece of lumber 24 ft. $\times$ 1 ft. $\times$ 9 in.?

CORD WOOD



Cord foot

Cord

A cord of wood is as much wood as is contained in a pile measuring 4 ft. $\times$ 4 ft. $\times$ 8 ft.

A cord = 128 cu. ft. in space.

The wood is piled as it comes, and the space not actually taken by wood counts just as much as the solid wood.

A cord foot is 4 ft. $\times$ 4 ft. $\times$ 1 ft.

A cord foot = 16 cu. ft. of space.

1. How many cord feet are there in a cord?
2. Will's father bought 20 cords of wood. If this was piled 4 ft. wide and 8 ft. high, how long would the pile be?
3. What part of a cord is 2 cord feet? 3 cord feet?
4. A pile of wood 4 ft. $\times$ 12 ft. $\times$ 12 ft. was offered to John Douglas at \$5 a cord. He found the amount of the bill in this way:

$$4 \times 12 \times 12 = 4 \times 3 \times 4 \times 3 \times 4 = 4 \times 4 \times 4 \times 3 \times 3 = \frac{1}{2} \text{ cord} \times 9 = \frac{9}{2} \text{ cords} = 4\frac{1}{2} \text{ cd.} \quad \$5 \times 4\frac{1}{2} = \$20\frac{1}{2} = \$20.50.$$

Can you follow these steps?

ADDITION

In addition, the numbers to be added are called **addends**.

Tell the **addends**, the **subtrahends**, the **minuends**, and **differences**; the **multiplicands**, the **multipliers**, and the **products**; the **dividends**, the **divisors**, and the **quotients**; the **ratios** and the **fractions** in the following questions:

1. A farmer bought 6 cows at \$30 each from a trader, 4 cows at \$32 each from a neighbor, and 2 cows at \$75 each from a fancy stock farmer. He then had in all 15 cows. How many cows had he to begin with? How much did all the new cows cost him?

2. In a regiment of 708 men there were 120 veterans, and the rest were raw recruits. The average age of the veterans was 38 yr., while that of the recruits was 15 yr. less. How many raw recruits were there? What was their average age?

3. A boy bought two saws, a hammer, a hatchet, a file, a plane, a try-square, two chisels, a vise, and a 2-foot rule. These cost respectively \$1.50 and \$1 for the saws; 75¢, 90¢, 20¢, \$1.35, 40¢, for the next tools; 40¢ and 30¢ for the chisels; \$2½ and 30¢ for the last articles. How many tools did he buy? At what total cost? At the same general prices, how much would 33 tools have cost?

4. One girl had 7 paper dolls, another 3 times as many, another $\frac{1}{7}$ as many as the second, and a fourth as many as the second and third together. How many paper dolls had they altogether? If they could have sold them at 2¢ each, how much would each of them have received for her dolls? How much would all have received for all the dolls?

GENERAL MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

1. Read the multiplication table of each number, beginning $2 \times 1 = 2$, $2 \times 2 = 4$, $2 \times 3 = 6$, and so through 2's; then $3 \times 1 = 3$, and so on through all numbers.

2. Read the division facts in this way, beginning $4 \div 2 = 2$, $6 \div 2 = 3$, $8 \div 2 = 4$, and so through the first column; then $6 \div 3 = 2$, $9 \div 3 = 3$, $12 \div 3 = 4$; and so on through all the numbers.

3. Read the columns down, 2, 4, 6, 8, and so on; 3, 6, 9, 12, and so on, telling in what multiplication table we find these numbers.

4. What numbers multiplied together give 144, 132, 121, 120, 110, 108, 100, 99, and so on through all these numbers?

DIVISION

If we divide one number by another, we find how many times the divisor is in the dividend. Let us divide

10 into 2's: $\begin{array}{ccccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} = \begin{array}{|c|c|c|c|} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} = 5 \text{ 2's.}$

Dividing is separating into parts. When we divide 48 by 12, we find 4 parts of 12 units each.

$12 + 12 + 12 + 12 = 48$	12	It is much easier to multiply two numbers than to add many. Division is also easier than many subtractions.
	12	
$48 - 12 - 12 - 12 - 12 = 0$	12	
	$\frac{12}{48}$	

If we wished to separate 360 into parts of 10 each, how hard it would be to find the answer in this way: $360 - 10 = 350$, $350 - 10 = 340$, $340 - 10 = 330$, etc.; and then to count all the 10's! Instead, we can write $\begin{array}{r} 10 \overline{)360} \\ \underline{36} \end{array}$. There are 36 10's in 360.

DIVIDING BY 10, 100, 1000

1. Divide 8000 by 100.

$\begin{array}{r} 100 \overline{)8000} \\ \underline{80} \end{array}$ Cancel two zeros in 100 and 8000.
80 8000 = 80 100's.

2. Divide six millions by three millions.

Cancel the millions. $\frac{6}{3} = 2$.

3. If New York City had in 1900 about $3\frac{1}{2}$ millions of people, and the whole United States had in 1800 7 millions, what was the ratio of population in all the United States then to New York one century later?

Divide: 4. 64,000 by 8000. 5. 15,000,000 by 3,000,000.

DIVISION

Division finds how many times one number is contained in another.

The number to be divided is called the **dividend**.

The number we divide by is called the **divisor**.

The result obtained by division is called the **quotient**. It shows how many times the divisor is contained in the dividend.

When the dividend does not contain the divisor an exact number of times, the part of the dividend left undivided is called the *remainder*, which is always less than the divisor.

The sign of division, $\div$, shows that the number before it is to be divided by the number after it. Thus,

$$10 \div 5 = 2. \quad \text{Ten divided by five is two.}$$

Division is also indicated by writing the dividend above a line and the divisor below it; thus, $\frac{10}{5} = 2$.

The sign $)$ also is used to indicate division. Thus $5)10$ shows that 10 divided by 5 equals 2. We sometimes show by this form, $8 \overline{)64}$, that division is desired.

Proof. Multiply the quotient by the divisor and add the remainder, if any. If the result equals the dividend, the work is correct.

$$\begin{array}{rcll} 8 \overline{)65} & 8 \times 8 = 64 + 1 & 8 \overline{)216} & 27 \text{ quotient} \\ 8 + 1 & & 27 & \times 8 \text{ divisor} \\ & & \hline & & 216 & \text{dividend} \end{array}$$

Divide 433 by (1) 5; (2) 7; (3) 19.

By introducing the fraction we may secure exact division always. To do this we treat the remainder as a fraction of which the divisor is the denominator and the remainder the numerator.

$$\begin{array}{r} 8 \overline{)65} \\ 8 \frac{1}{8} \end{array} \quad 8 \times 8 \frac{1}{8} = (8 \times 8) + (8 \times \frac{1}{8}) = 64 + 1 = 65.$$

SHORT DIVISION

We know now the multiplication tables of all numbers from 2 to 12. We call division by these numbers **short division** because there is very little writing to do.

$$\begin{array}{r} 8 \overline{)89616} \\ \underline{11202} \end{array} \quad \begin{array}{r} 8 \overline{)192168} \\ \underline{24021} \end{array} \quad \begin{array}{r} 9 \overline{)90189} \\ \underline{10021} \end{array} \quad \begin{array}{r} 7 \overline{)1109964} \\ \underline{1585664} \end{array}$$

The first of these problems would be in **long division** if we put down all the processes in full.

$$\begin{array}{r} 8 \overline{)89616} (11202 \\ \underline{8} \quad 1 \times 8 = 8 \\ \underline{09} \\ \underline{08} \quad 1 \times 8 = 8 \\ \underline{16} \\ \underline{16} \quad 2 \times 8 = 16 \\ \underline{01} \\ \underline{00} \quad 0 \times 8 = 0 \\ \underline{16} \\ \underline{16} \quad 2 \times 8 = 16 \end{array}$$

Divide each dividend in each group by each divisor.

GROUP I		GROUP II	
Dividend	Divisor	Dividend	Divisor
1. 11,680	(a) 2	5. 732,119	(e) 6
2. 75,010	(b) 3	6. 732,107	(f) 7
3. 80,171	(c) 4	7. 11,988,959	(g) 8
4. 110,519	(d) 5	8. 11,010,989	(h) 9

	Dividend	Divisors
9-10-11-12.	510,384	÷ 7, 8, 9, 11 =
13-14-15-16.	3,627,089	÷ 6, 7, 9, 12 =
17-18-19-20.	1,084,608	÷ 12, 9, 6, 3 =
21-22-23-24.	1,018,193	÷ 11, 9, 7, 5 =

LONG DIVISION

1. Divide 4365 by 14.

14)4365(311 $\frac{11}{14}$ quotient

$$\begin{array}{r}
 42 \\
 \underline{16} \\
 14 \\
 \underline{25} \\
 14 \\
 \underline{11}
 \end{array}$$

14 is contained in 43 hundreds 3 (hundreds) times and 1 hundred remaining. We write 3 in hundreds' place in the quotient. 1 hundred and 6 tens = 16

tens. 14 is contained in 16 tens 1 (ten) times and 2 (tens) remaining. We write 1 in tens' place. 2 tens and 5 units = 25 units. 14 is contained in 25 units 1 (unit) time and 11 units remaining. We write 1 in units' place and put 11 over 14, with a line between the two figures to show that the 11 is still to be divided by 14.

The process of **long division** is the same as that of short division, but in the former the work is written out in full. When the divisor is larger than 12, we generally employ the method of **long division**.

2. Divide 17,092 by 23.

23)17092(743 $\frac{3}{23}$

$$\begin{array}{r}
 161 \\
 \underline{99} \\
 92 \\
 \underline{72} \\
 69 \\
 \underline{3}
 \end{array}$$

Proof: 743 . . quotient

23 . . divisor

$$\begin{array}{r}
 2229 \\
 1486
 \end{array}$$

3 . . remainder

17092 . . dividend

3. Divide 96,875 by 57.

4. Divide 105,290 by 75.

5. Divide 41,278 by 24.

6. Divide 86,397 by 17.

7. Divide 192,689 by 33.

8. Divide 207,175 by 25

9. 66)432,824(

10. 59)127,567(

11. 37)368,129(

12. $82932 \div 24 = ?$ 13. $96528 \div 99 = ?$ 14. $125625 \div 25 = ?$

LONG DIVISION

GROUP I

Divide	I.	27573	(a)	13
each	II.	3378	(b)	16
dividend	III.	9034	(c)	17
by each	IV.	3978	(d)	19
divisor	V.	7605	(e)	18

GROUP II

Divide	VI.	910,372	(f)	93	Why will there be twenty-five different quotients in each of these three groups?
each	VII.	588,168	(g)	84	
dividend	VIII.	11,101	(h)	101	
by each	IX.	98,200	(i)	99	
divisor	X	64,466	(j)	94	

GROUP III

Divide	XI.	92,483	(k)	23
each	XII.	78,104	(l)	26
dividend	XIII.	13,527	(m)	27
by each	XIV.	13,066	(n)	65
divisor	XV.	32,216	(o)	46

1. An excursion steamer is licensed to carry 425 passengers. How many trips must it make to carry 3825?

2. A boat licensed to take no more than 125 passengers a trip has 2337 persons to transport. How many trips must it make to do so? If it takes 125 passengers on each trip but the last one, how many passengers will it carry on the last trip?

3. A man left property valued at six hundred thousand dollars to be divided equally among his eleven children, after giving his wife one third of all the property. How many dollars should be given to each heir?

REVIEW

1. At \$3 a dozen, how many chickens can be bought for \$15.75?
2. A man bought 6 house lots for \$5580. What was the average cost of each lot?
3. When land is worth \$76 an acre, how many acres can one buy with \$12,312?
4. A person's income is \$1568 a year. What is his income a week?
5. A newsboy earns \$19 a month. How many years will it take him to earn \$1368?
6. If one door knob takes half a pound of metal, how many door knobs will 2528 ounces of metal make?
7. There are 196 pounds of flour in a barrel. How many barrels can be packed from 18,816 pounds of flour?
8. If the yield of 110 sugar canes is 6 pounds of sugar, how many pounds of sugar will 1,233,210 canes yield?
9. In a regiment of 1280 men there were 64 officers. How many privates were there to each officer?
10. In how many days will a traveler, driving at the rate of six miles an hour for seven hours a day, complete a journey of five hundred miles?
11. For one page of a book containing 36 lines, 2052 letters were used. How many letters, on an average, were there in each line?
12. If you had to divide 72 dozen oranges among 430 children, how many whole oranges could you give to each child? How many oranges would you have left?
13. The circumference of the globe at the equator is about 24,902 miles. How many hours would it take a train, traveling 28 miles per hour, to go around the globe?
14. Draw a hexagon. Divide it into six triangles.

A.	B.	C.	D.	E.
87406	92674	25043	179122	262083
41299	27049	97069	7626	198568
47208	28372	81216	6293	4318
71615	37041	90414	7404	39665
72428	49741	19216	30359	26881
28907	57214	72859	16660	34705
27426	92742	87604	80901	95299
87489	17618	27042	17255	80597
97206	21859	72687	32543	53806
25412	51814	18972	56063	8624
70412	59261	59876	40022	35077
62084	40216	32018	17548	29997
21642	42673	54301	16147	39744
81697	57274	71215	38556	44305
72204	87614	21441	234882	8735
<u>41278</u>	<u>41219</u>	<u>60594</u>	<u>152526</u>	<u>169220</u>

Proofs. Beginning with the unit's column, add each column, up and down. Or, separate the numbers into two or more sets, find the sum of each set, and add the sums.

107485075	107485075
870302861	870302861
73628874	73628874
668279	<u>1051416810</u> first partial sum.
47538234	
267849026	668279
486792836	47538234
9563748	<u>267849026</u>
6473928	<u>316055539</u> second partial sum.
<u>1870302861</u> total sum.	486792836
1051416810	9563748
316055539	6473928
<u>502830512</u>	<u>502830512</u> third partial sum.

REVIEW

Divide:

- | | |
|------------------------------|---------------------------|
| 1. 463,750 by 7, 8, 9, 11. | 5. 94,328 by 6, 8, 9, 10. |
| 2. 4,200,560 by 5, 7, 8, 10. | 6. 43,272 by 3, 6, 9, 12. |
| 3. 7,200,045 by 9, 8, 7, 6. | 7. 377,424 by 4, 6, 7, 8. |
| 4. 4,704,091 by 7, 11, 9, 5. | 8. 975,216 by 5, 6, 8, 9. |

Multiply:

- A. 5019 by 14, 15, 18, 21, 27, 32, 36, 42.
- B. 67,895 by 56, 63, 72, 84, 96, 77, 45, 54.
- C. 75,098 by 16, 20, 30, 40, 50, 60, 70, 80.
- D. 97,014 by 27, 72, 63, 36, 54, 45, 88, 99.
- E. 80,694 by 96, 60, 48, 84, 121, 132, 144.
- F. 97,065 by 132, 121, 108, 99, 120, 100, 110.

I. In the problems 1 to 8 in division, prove the quotients correct by multiplying them by the divisors, and seeing if you get the dividends.

II. In the problems A to F in multiplication, prove the products correct by dividing them by the multipliers, and seeing if you get the multiplicands.

III. Do you see now the value of knowing thoroughly the multiplication tables? Are they as useful in division as in multiplication? Why?

PUNCTUATION

If it helps you to be accurate, write 463,750 with the comma, to show thousands, and 4,200,560 with the commas, to make the thousands and millions plain.

MAKING PROBLEMS

Using such numbers as these on this page, make oral or written problems like those in different parts of this book.

REVIEW

1. A dealer sold 19 lambs for \$76. How much apiece on an average did they cost the buyer?

2. A farmer secured 198 bushels of wheat from a seven-acre lot. How many bushels to an acre did the lot yield?

3. What is the ratio of 2 yd. to 3 in.?

4. How many times must we move and place a two-foot rule to measure two rods?

5. What is the ratio of a prism containing 32 cu. in. to another containing 640 cu. in.?

6. What is the ratio of a prism containing one cubic foot to another containing four hundred thirty-two cubic inches?

7. A farmer sold his farm of 137 acres for \$8631. How much an acre did he get for the farm?

8. How many times is \$17 contained in \$11,917?

9. What sum of money taken 17 times will amount to \$11,917?

10. Two dozen hens each laid in one year 12 dozen eggs. How many eggs in all were laid?

12 dozen make 1 gross.

11. A farmer cut in his woods one winter 125 cords. If the wood was piled 4 ft. wide and 4 ft. high, how long was the pile? How many cubic feet are there in 125 cords? This number divided by $4 \times 4 =$ what?

12. Which is greater, 5 cu. ft. or 1 cd.? By how much?

13. A grocer bought 3 gross of eggs at 10¢ a dozen. How much did he pay?

MONEY

What is the smallest coin we use?

What is the value of a nickel? dime? quarter?

We write seven dollars sixty-three cents, \$7.63; twenty-one dollars fourteen cents, \$21.14; two dollars four cents, \$2.04; eleven dollars ninety cents, \$11.90.

$\frac{1}{10}$ of a cent is called a mill. Mills are written at the right of cents: we write three cents three mills, \$.033; one dollar sixteen cents nine mills, \$1.169; twenty dollars five mills, \$20.005.

10 mills make 1 cent

100 cents make 1 dollar

Mills occupy the third place at the right of the decimal point, and cents the first two places.

The United States government issues gold coins.

quarter eagle = \$2.50 eagle = \$10.00

half-eagle = \$5.00 double eagle = \$20.00

What bills are issued by government? What was the value of the largest bill you have ever seen? What was the value of the smallest?

1. What three coins together make 25¢?
2. What coins together make 11¢? 16¢? 19¢? 31¢? 49¢? 51¢? 65¢? 75¢?
3. How many nickels would you give for $\$ \frac{1}{2}$? How many dimes? How many quarters?
4. What is the smallest silver coin called? What is the value of our largest silver coin?
5. How many mills are there in a half dollar?
6. Silver is worth about 56¢ an ounce. A silver dollar weighs $\frac{7}{8}$ of an ounce. What is the value of the silver in a dollar?

MONEY

$\$ \frac{1}{4}$ expressed decimally is \$.25. $\$ \frac{1}{2}$ = how many cents?

1. Write decimally $\$ \frac{1}{2}$, $\$ \frac{3}{4}$, $\$ \frac{1}{5}$, $\$ \frac{7}{10}$, $\$ \frac{1}{20}$.

2. Read and add :

\$ 1.46	\$ 19.01	\$ 77.77	\$.05
2.00	19.011	10.01	.005
20.00	60.03	44.04	.50
200.00	113.13	266.666	50.051
5.00	200.20	1.001	1.

3. Is there any difference between $\$ 1\frac{1}{2}$ and \$1.50?

4. Is there any difference between $\$ \frac{1}{10}$ and 1 dime?

5. Write eight dollars two dimes and two cents.

6. How many dollars are there in \$39.468? How many dimes or *tenths* of a dollar? How many cents or hundredths of a dollar? How many mills or thousandths?

7. Write fifty cents in two ways.

8. How many dollars and cents are there in :

330¢?	275¢?	122¢?	196¢?	288¢?	110¢?
108¢?	222¢?	770¢?	704¢?	636¢?	565¢?

9. Write decimally :

6 dollars 2 dimes 8 cents 8 mills
 22 dollars 7 dimes 3 cents 1 mill
 303 dollars 0 dimes 1 cent 0 mills
 72 dollars 6 dimes 0 cents 0 mills
 66 dollars 6 dimes 6 cents 6 mills

10. How many $\$ \frac{1}{2}$ are there in \$6? \$20? \$22? \$30?

11. How many \$.25 are there in \$1? \$2? \$4? \$8? \$10?

12. How many cents are there in two half dollars?
 in three quarters?

FRACTIONS

1. How many cents are there in $\frac{3}{4}$ of 100 cents? in $\frac{1}{4}$ of 300 cents? Is there any difference between $\frac{3}{4}$ of 1 and $\frac{1}{4}$ of 3?

2. What is $\frac{1}{4}$ of 8? $\frac{3}{4}$ of 8? $\frac{1}{4}$ of 24? $\frac{3}{8}$ of 6? $\frac{1}{8}$ of 12? $\frac{4}{5}$ of 10? $\frac{1}{5}$ of 40?

3. How many fourths are there in 1? in 2? in $2\frac{1}{4}$? in $3\frac{3}{4}$? in $6\frac{1}{4}$? in 12? in $12\frac{3}{4}$?

4. How many fifths are there in 1? in 3? in $3\frac{1}{5}$? in 5? in $5\frac{3}{5}$? in $10\frac{1}{5}$?

5. How many sixths are there in 1? in 2? in 4? in $5\frac{1}{6}$? in $9\frac{5}{6}$? in $11\frac{4}{6}$?

6. How many sevenths are there in 1? in 3? in 5? in $5\frac{1}{7}$? in $6\frac{2}{7}$? in $1\frac{1}{7}$?

7. How many eighths are there in 1? in 2? in $4\frac{1}{8}$? in $5\frac{3}{8}$? in $6\frac{1}{8}$?

8. Show by drawings that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

9. Show also that $\frac{2}{3} = \frac{1}{\frac{3}{2}}$ and that $\frac{3}{6} = \frac{1}{2}$.

REDUCTION OF FRACTIONS

Since $\frac{1}{2} = \frac{2}{4}$, and $\frac{1}{3} = \frac{2}{6}$, then multiplying both the numerator and the denominator of a fraction does not change its value, for while each fractional part is made smaller the number of parts taken is so increased that the ratio of the numerator to the denominator remains the same.

10. What is the ratio of 1 to 2? of 2 to 4?

11. What is the ratio of 1 to 3? of 3 to 6?

Multiplying both the numerator and the denominator of a fraction is one way of *reducing* a fraction, or changing its terms without changing its value.

LEAST COMMON MULTIPLE

A **multiple** of a number is a number exactly divisible by that number.

15, 20, and 45 are multiples of 5.

A **common multiple** of two or more given numbers is any number exactly divisible by each of them.

45 is a common multiple of 3, 5, and 9.

The **least common multiple** of two or more numbers is the least number that is exactly divisible by each of them.

84 is the least common multiple of 3, 7, 14, and 21.

A multiple of a number contains all the prime factors of that number. A multiple of two or more numbers contains all the prime factors of those numbers.

The least number that exactly contains all the prime factors of two or more numbers must be the least common multiple of those numbers.

1. Find the L. C. M. of 12, 20, 30, 54.

2	12	20	30	54	L. C. M. = $2 \times 2 \times 3 \times 5 \times 9 = 540$
2	6	10	15	27	
3	3	5	15	27	
5	1	5	5	9	
	1	1	1	9	

The L. C. M. of two or more numbers must contain all the different prime factors in the numbers; and if a prime factor is repeated in any of them, it must be repeated in the L. C. M. as many times as it is repeated in that number in which it is most often repeated, and no more.

2. Find the L. C. M. of 18, 24, 27, 45.

$$18 = 2 \times 3 \times 3$$

$$24 = 2 \times 2 \times 2 \times 3 \quad \text{The required L. C. M. is } 2 \times 2 \times 2$$

$$27 = 3 \times 3 \times 3 \quad \times 3 \times 3 \times 3 \times 5 = 1080.$$

$$45 = 3 \times 3 \times 5$$

GREATEST COMMON DIVISOR

A **common divisor** of two or more numbers is any number greater than 1 that exactly divides each of them.

The **greatest common divisor** of two or more numbers is the greatest number that exactly divides each of them.

Numbers that have no common divisor are said to be **prime** to one another.

When numbers have been resolved into their *prime factors*, their G. C. D. can be obtained by inspection.

1. Find the G. C. D. of 36, 42, and 84.

$$36 = 2 \times 2 \times 3 \times 3$$

$$42 = 2 \times 3 \times 7 \quad \text{G. C. D. is } 2 \times 3.$$

$$84 = 2 \times 2 \times 3 \times 7$$

The G. C. D. required is 6; for each number is divisible by 2 and by 3, and there is no other prime factor common to all three numbers.

2. Find the G. C. D. of :

a. 8 and 14. e. 12 and 30. i. 16, 64, 256, 1024.

b. 40 and 60. f. 36 and 90. j. 110, 140, and 680.

c. 48 and 144. g. 7, 14, 21. k. 24, 51, 105, 729.

d. 15, 27, 105. h. 32, 48, 128. l. 1326, 3094, 4420.

3. What is the G. C. D. of 1365 and 1995 ?

$$\begin{array}{r|l} 3 & 1365 \\ 5 & 455 \\ 7 & 91 \\ & 13 \end{array}$$

$$\begin{array}{r|l} 3 & 1995 \\ 5 & 665 \\ 7 & 133 \\ & 19 \end{array}$$

The prime factors of 1365 are 3, 5, 7, 13.

The prime factors of 1995 are 3, 5, 7, 19.

$$3 \times 5 \times 7 = 105 = \text{G. C. D. of 1365 and 1995}$$

4. Find the G. C. D. :

- a. 6, 12, and 30. d. 36, 63, and 99. g. 25, 35, and 70.
 b. 40, 64, and 84. e. 12, 18, and 72. h. 72 and 168.
 c. 15, 25, and 30. f. 85 and 95. i. 28, 42, and 70.

5. Find the G. C. D. of 304 and 1072.

TEACH

$$304 \overline{)1072} (3$$

$$\begin{array}{r} 912 \\ \underline{160} \end{array}$$

$$304 \overline{)160} (1$$

$$\begin{array}{r} 160 \\ \underline{144} \end{array}$$

$$144 \overline{)160} (1$$

$$\begin{array}{r} 144 \\ \underline{144} \end{array}$$

$$16 \overline{)144} (9$$

$$\begin{array}{r} 144 \\ \underline{144} \end{array}$$

We divide 1072 by 304, the remainder is 160; we then divide 304 by 160, the remainder is 16; lastly, we divide 144 by 16, with no remainder.

$$16 \overline{)304} (19$$

$$16 \overline{)1072} (67$$

We divide the greater of the two given numbers by the smaller, then the divisor by the remainder; next the last divisor by the new remainder, and so on until there is no remainder. The last divisor will be the G. C. D. required.

The G. C. D. of dividend and divisor is also the G. C. D. of divisor and remainder.

If the last divisor is unity, the given numbers have no common measure, that is, they are prime to each other.

6. Find the G. C. D. of :

WRITE

- | | |
|-----------------------|----------------------|
| a. 1024 and 2240. | b. 11256 and 19899. |
| c. 1902 and 24409. | d. 5161 and 7755. |
| e. 1624 and 14500. | f. 87147 and 178871. |
| g. 8393 and 4609. | h. 1261 and 663. |
| i. 714 and 1176. | j. 918 and 1347. |
| k. 219 and 11476. | l. 187 and 255. |
| m. 194706 and 289913. | n. 1914 and 35786. |
| o. 2925 and 29484. | p. 21671 and 22111. |
| q. 27525 and 1725. | r. 82159 and 582. |

G. C. D. AND L. C. M.

1. Find the greatest common divisor of 36, 60, 96, and 144.

2	36	60	96	144
2	18	30	48	72
3	9	15	24	36
	3	5	8	12

The common divisors are 2, 2, 3.

$$2 \times 2 \times 3 = 12$$

The greatest common divisor is 12.

2. Find the least common multiple of 36, 60, 96, and 144.

The divisors common to two or more of these numbers are 2, 2, 3, 2, 3, 2. The prime dividends are 5, 2.

$$2 \times 2 \times 3 \times 2 \times 3 \times 2 \times 5 \times 2 = 1440$$

$$4 \quad 12 \quad 24 \quad 72 \quad 144 \quad 720 \quad 1440$$

2	36	60	96	144
2	18	30	48	72
3	9	15	24	36
2	3	5	8	12
3	3	5	4	6
2	1	5	4	2
	1	5	2	1

To find the least common multiple we extend the process of finding the common divisors and multiply all the prime factors together.

PROOF OF 1

12)	36	60	96	144
	3	5	8	12

No larger number than 12 will divide all the numbers, since the dividends of 36 and 60 by 12 as divisor are prime to each other.

PROOF OF 2

36)1440(40	60)1440(24	96)1440(15	144)1440(10)
144	120	96	
<u>0</u>	<u>240</u>	<u>480</u>	
<u>0</u>	<u>240</u>	<u>480</u>	

No smaller number than 1440 will contain all these numbers, since 40, 24, 15, and 10 have no common factor.

3. Find the G. C. D. and L. C. M. of 16, 32, 64, 96.

4. Find the G. C. D. and L. C. M. of 75, 125, 200, 250.

[illegible]

1. $\frac{1}{2} = \frac{?}{4} = \frac{?}{8} = \frac{?}{16} = \frac{?}{32}$.

$$8. \quad \begin{array}{cccc} 3 & 2 & 2 & 2 \\ 4 & 8 & 16 & 32 \end{array}$$

2. $\frac{1}{4} = \frac{?}{8} = \frac{?}{16} = \frac{?}{32} = \frac{?}{64}$.

6. $\frac{3}{8} = \frac{?}{16} = \frac{?}{32} = \frac{?}{64}$

3. $\frac{1}{8} = \frac{?}{16} = \frac{?}{32} = \frac{?}{64} = \frac{?}{128}$.

7. $\frac{5}{8} = \frac{?}{16} = \frac{?}{32} = \frac{?}{64}$

4. $\frac{1}{16} = \frac{?}{32} = \frac{?}{64} = \frac{?}{128} = \frac{?}{356}$.

8. $\begin{matrix} 7 & ? & ? & ? \\ 8 & 16 & 32 & 64 \end{matrix}$

$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$
$\frac{1}{9}$			$\frac{1}{9}$			$\frac{1}{9}$		$\frac{1}{6}$			$\frac{1}{6}$			$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	
$\frac{1}{3}$								$\frac{1}{2}$						$\frac{1}{5}$									

9. $\frac{1}{3} = \frac{3}{9} = \frac{2}{6} = \frac{5}{15}$.

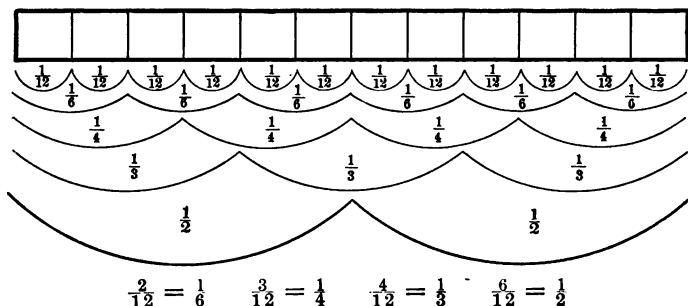
11. $\frac{2}{9} \cdot \frac{9}{27}$

10. $\frac{1}{3} = \frac{2}{27} = \frac{2}{18} = \frac{2}{30}$.

12. $\frac{2}{15} = \frac{?}{30}$

13. Write in order of size: $2^{\frac{1}{27}}$, $1^{\frac{1}{18}}$, and $2^{\frac{1}{9}}$.

TWELFTHS



1. If anything is divided into any number of parts, what ratio of the numerator to the denominator always indicates the whole thing?

2. If we change the numerator of a fraction by multiplying it, why must we also always multiply the denominator by the same number?

3. In these fractions, what is the ratio of the denominators to the numerators : $\frac{3}{12}$, $\frac{1}{4}$, $\frac{6}{24}$?

4. How many eighths are there in one fourth? how many twelfths? how many sixteenths?

5. $\frac{4}{6} = \frac{?}{12}$ $\frac{3}{12} = \frac{?}{4}$ $\frac{1}{3} = \frac{?}{12}$ $\frac{9}{12} = \frac{?}{4}$

6. Draw a figure representing $\frac{1}{12}$. If each twelfth should represent a sixth of the figure, how many wholes would this figure represent?

7. If each twelfth should represent a fourth, how many wholes would we have?

8. $\frac{1}{3} = ?$ $\frac{1}{2} = ?$ $\frac{1}{8} = ?$ $\frac{1}{9} = ?$

REDUCING FRACTIONS

Sometimes it is convenient for us in working with a fraction to have its denominator larger. This means that the unit is to be divided into more equal parts.

The real value of the fraction remains the same.



$$\frac{1}{4} = \frac{2}{8}$$



To reduce fractions to higher terms, we multiply both the denominator and the numerator by a number larger than one. Higher terms mean more equal parts.

1. By what number is $\frac{1}{4}$ multiplied in reducing it to $\frac{2}{8}$?

2. Reduce $\frac{2}{5}$ to 15ths. $15 = 5 \times 3$. $\frac{2}{5} = \frac{6}{15}$.

To make the denominator 15, we multiply it by 3. If we multiply the numerator also by 3, we do not change the value of the fraction.

3. Reduce $\frac{7}{17}$ to 68ths.

$$\begin{array}{r} 17 \overline{)68} (4 \\ \underline{68} \end{array} \quad \frac{7}{17} \times \frac{4}{4} = \frac{28}{68}$$

$$\text{Proof: } \begin{array}{r} 4 \overline{)28} = 7 \\ 4 \overline{)68} = 17 \end{array}$$

4. Reduce $\frac{11}{5}$ to a fraction whose denominator is 90.

5. Reduce $\frac{19}{7}$ to a fraction whose denominator is 54.

6. Change 9 to a fraction whose denominator is 3.

$$1 = \frac{3}{3} \quad 9 = 1 \times 9 \quad \frac{3}{3} \times 9 = \frac{27}{3} \quad 9 = \frac{27}{3}$$

7. Change 9 to a fraction whose denominator is 11.

Reduce :

8. $\frac{5}{8}$ to 72ds. 9. $\frac{2}{9}$ to 49ths. 10. $\frac{4}{9}$ to 27ths.

11. $\frac{2}{9}$ to 81sts. 12. $\frac{12}{5}$ to 150ths. 13. $\frac{3}{10}$ to hundredths.

14. $\frac{4}{7}$ to 63ds. 15. $\frac{16}{4}$ to 128ths. 16. $\frac{5}{100}$ to thousandths.

REVIEW

1. If a clock ticks 3600 times an hour, how many times will it tick in two days of 24 hours each?

2. Divide a tax of \$53,400 equally among 12 towns.

3. The quotient is 40, the divisor is 364, and the remainder is 120. What is the dividend?

4. A man's salary is \$780 per annum. What is it per month? per week?

5. The sum of \$4824 is to be raised from 12 counties, in each of which are 6 towns. How much must be raised in each town?

6. Twenty-one casks contain 10,941 herrings. How many herrings are there on the average in each cask?

7. The earth moves around the sun at the rate of 66,600 miles an hour. How many miles is that per minute?

8. Reduce $\frac{4}{7}$ ths to 49ths, and add $\frac{6}{49}$ ths. What is the result?

9. In round numbers what is the earth's circumference? How many days would it take to travel around the earth, going at the rate of 125 miles a day?

10. Reduce to smaller denominations: $\frac{8}{14}$, $\frac{6}{18}$, $\frac{10}{28}$, $\frac{12}{36}$.

11. There are eight houses in a village, and each house has seventeen windows. How many windows are there in all the houses? At 6 panes to a window, how many are there in all?

12. A man who had 6 cords of wood made a pile 8 ft. high and 12 ft. wide. How long was it?

13. A public park contained 919,960 sq. ft. It was 155 ft. wide. How long was it?

14. A boy ran 1000 yd. Was this more than $\frac{1}{2}$ mi.?

REVIEW

1. A boy collected 125 foreign postage stamps at an average cost of 4¢ each. How much money did he spend?

2. 5 of the stamps cost \$1.40. How much in all did the other stamps cost?

3. What was the average cost of the 5 stamps? What was the average cost also of the 120 other stamps?

4. Count by 16 to 200.

5. Read:

\$782 039 648 521; 84 326 800 000; 100 600 000 000.

6. The wealth of the United States in 1900 was about ninety billion dollars. How many figures including zeros must we write to express this?

7. If the average wealth of all the persons in the United States in 1900 was \$1200, what was the average wealth of the average family of five persons?

8. How do we multiply by 10, 100, 1000 most easily?

9. What fractional part of a dollar is 5¢? 10¢? 25¢?

10. What is 14×12 ? 15×11 ? 17×12 ? 16×11 ?

11. What is the volume of a prism 8 ft. $\times$ 5 ft. $\times$ 5 ft.?

12. What is the volume of a prism $12\frac{1}{2}$ ft. $\times$ $\frac{1}{4}$ ft. $\times$ 2 ft.?

13. What is the ratio of the volume of the prism in 12 above to that of the prism in 11?

14. What fraction with a smaller denominator is equivalent to $\frac{3}{24}$? $\frac{2}{16}$? $\frac{4}{20}$?

15. Give the tables for Long Measure, Square Measure, and Cubic Measure.

16. Tell the meanings of: area, scale, addend, minuend, subtrahend, multiplicand, dividend, quotient, fraction, ratio, per cent, prism, cubic measure, cord.

FACTORS

The **factors** of a number are the numbers that multiplied together produce the number.

The factors of 12 are 4 and 3, 6 and 2, or 2, 2, and 3; for $4 \times 3 = 12$, $6 \times 2 = 12$, or $2 \times 2 \times 3 = 12$.

As the factors, 2, 2, 3, 4, and 6, will exactly divide 12, those numbers are also **divisors** of 12.

An **integer**, or an integral number, is a whole number. 1, 6, 8, 22, are integers.

$6\frac{1}{2}$, $3\frac{7}{8}$, $\frac{3}{4}$, $\frac{7}{11}$, $12\frac{1}{9}$, are not integers, but **mixed numbers**, since they are made up of integers and fractions.

A **prime number** is a number which has no integral factor except itself and 1.

3, 5, 7, 11, 13, 19, are prime numbers.

Numbers are prime to one another when no number but 1 will exactly divide each of them. Thus, 3 and 5; 4 and 7; 8 and 19; 21 and 2; 6, 11, 17, and 25, are prime to each other.

A **prime factor** is a factor that is a prime number.

2, 2, and 3 are the prime factors of 12. The prime factors of 18 are $3 \times 2 \times 3$. The prime factors of 24 are $2 \times 2 \times 2 \times 3$.

A number that is a factor of two or more numbers is called a **common factor**, or a common divisor of the numbers.

2 is a common factor, or a common divisor, of 6, 12, 14, 18, 20, 22, because it will exactly divide each of them.

A **composite number** is a number that has integral factors besides itself and 1.

$4 = 2 \times 2$, $9 = 3 \times 3$, $14 = 2 \times 7$, $39 = 3 \times 13$, $55 = 5 \times 11$,
 $100 = 2 \times 2 \times 5 \times 5$, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

FACTORS

1. Name the eight factors of 24.
2. Give all the factors of 4, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30.
3. What is a factor? a prime number? a composite number? Give two numbers of each kind.
4. What number is always a factor of even numbers?
5. Is there any prime number above 2 which is not an odd number?
6. Find all the prime numbers from 3 to 53.
7. Name all the factors of 33, 35, 39, 42, 45, 50.
8. Name all the composite numbers from 48 to 60.
9. What prime factor is common to
6 and 9? 10 and 12? 15 and 20? 11 and 22?
16 and 24? 24 and 8? 30 and 35? 28 and 14?
10. What is the largest factor common to
12 and 24? 10 and 30? 20 and 36? 24 and 36?
36 and 27? 16 and 32? 26 and 13? 40 and 60?
11. What is a factor? A prime factor? A common factor? A composite number? Give an example of each.
12. What factor is common to the denominators of these fractions: $\frac{2}{3}$, $\frac{4}{21}$, $\frac{8}{9}$, $\frac{1}{24}$?
13. What factor is common to the numerators of these fractions: $\frac{12}{15}$, $\frac{8}{9}$, $\frac{24}{25}$, $\frac{60}{100}$?
14. $2 \times 3 \times 5 = ?$ $2 \times 2 \times 3 \times 3 = ?$ $3 \times 5 \times 7 = ?$
Why are these composite numbers?

TESTS FOR FACTORS

Any number is divisible by 2 if the unit figure is divisible by 2. Then 12, 16, 18, 152, 374, are divisible by 2, because 6, 8, 2, and 4 can be divided by 2.

Any number is divisible by 3 if the sum of its **digits**, or figures, is divisible by 3.

474 can be divided by 3, for its digits, added together, make a sum, 15, which is divisible by 3. ($4 + 7 + 4 = 15$.)

393 can be divided by 3, for $3 + 9 + 3 = 15$, a number divisible by 3.

1866 is divisible by 3, for $1 + 8 + 6 + 6 = 21$, a number of which 3 is a factor.

A number is divisible by 4 if the first two right-hand figures (units and tens) are divisible by 4.

144, 288, 824, can each be divided by 4, for 44, 88, and 24 are divisible by 4.

A number is divisible by 5 if its final digit is 5 or 0.

A number is divisible by 9 if the sum of its digits is divisible by 9.

738 is divisible by 9, for $7 + 3 + 8 = 18$, a number of which 9 is a factor. 117 is divisible by 9, for $1 + 1 + 7 = 9$, and $9 = 9 \times 1$.

1. Find the prime factors of 825.

2. What are the prime factors of 180?

$$\begin{array}{r} 5 \overline{) 825} \\ 5 \overline{) 165} \\ 3 \overline{) 33} \\ 11 \end{array}$$

$$825 = 5 \times 5 \times 3 \times 11$$

$$\begin{array}{r} 5 \overline{) 180} \\ 2 \overline{) 36} \\ 2 \overline{) 18} \\ 3 \overline{) 9} \\ 3 \end{array}$$

$$5 \times 2 \times 2 \times 3 \times 3 = 180$$

IMPROPER FRACTIONS

We know that any fraction in which the numerator is larger than the denominator is made up of two numbers, a whole number and a fraction, because an improper fraction takes more parts than a single unit is divided into. $\frac{8}{4}$ means 2 units, since 1 unit contains 4 fourths. $8 = 4 \times 2$.

A whole number is called an **integer**.

1. How many units are there in $1\frac{6}{4}$?

Since in 4 fourths there is 1 unit, in 16 fourths there are as many units as there are 4 fourths in 16 fourths: that is, 4 units. Hence $1\frac{6}{4} = 4$.

2. How many units are there in $2\frac{2}{3}$?

$2\frac{2}{3} =$ as many units as there are $\frac{2}{3}$ in $2\frac{2}{3}$, or $7\frac{1}{3}$ units.

3. Reduce $\frac{89}{12}$ to a mixed number.

$$\frac{89}{12} = 89 \div 12 = 7\frac{5}{12}$$

4. Change $1\frac{97}{37}$ to a mixed number.

$$\begin{array}{r} 37 \overline{)197} (5\frac{12}{37} \\ \underline{185} \\ 12 \end{array}$$

5. Reduce these fractions to whole or mixed numbers:

a. $1\frac{5}{4} = ?$ b. $5\frac{8}{8} = ?$ c. $4\frac{1}{9} = ?$ d. $4\frac{2}{7} = ?$ e. $1\frac{8}{7} = ?$
 f. $1\frac{2}{8} = ?$ g. $7\frac{5}{10} = ?$ h. $3\frac{11}{12} = ?$ i. $1\frac{5}{2} = ?$ j. $2\frac{6}{5} = ?$

6. Reduce to dollars and cents these numbers:

a. $\$ \frac{20}{3}$. b. $\$ \frac{67}{5}$. c. $\$ \frac{100}{12}$. d. $\$ \frac{50}{6}$. e. $\$ \frac{35}{8}$.
 f. $\$ \frac{2}{4}$. g. $\$ \frac{28}{3}$. h. $\$ \frac{25}{6}$. i. $\$ \frac{49}{8}$. j. $\$ \frac{14}{3}$.

IMPROPER FRACTIONS

The reason why fractions larger than a whole unit are called improper fractions is this: a fraction is an equal part of anything, and no part can ever be larger than the whole. It is not *proper*, because of these definitions, to call anything larger than a whole a fraction at all. But it is often convenient to do so, and we call numbers that are partly fractional *improper* fractions.

1. Reduce $3\frac{4}{5}$ to 5ths.

$$3\frac{4}{5} = \frac{15}{5} + \frac{4}{5} = \frac{19}{5}$$

Since there are 5 fifths in 1, in 3 there are 3 times 5 fifths, or 15 fifths; and $15 \text{ fifths} + 4 \text{ fifths} = \frac{19}{5}$.

2. Reduce $15\frac{6}{7}$ to 7ths.

$$\frac{15}{1} \times \frac{7}{7} = \frac{105}{7}$$

$$\frac{105}{7} + \frac{6}{7} = \frac{111}{7}$$

3. Reduce $20\frac{2}{13}$ to 13ths.

$$\frac{20}{1} \times \frac{13}{13} = \frac{260}{13}$$

$$\frac{260}{13} + \frac{2}{13} = \frac{262}{13}$$

4. How many 3ds are there in $3\frac{1}{3}$? $1\frac{2}{3}$? $2\frac{1}{3}$? $3\frac{2}{3}$?

5. How many 5ths are there in 1? in $1\frac{1}{5}$? in 2? in $1\frac{2}{5}$? in $2\frac{2}{5}$? in $4\frac{1}{5}$?

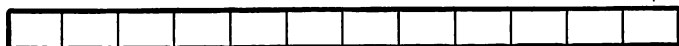
6. How many 7ths are there in 2? in 3? in $2\frac{3}{7}$? in $3\frac{6}{7}$? in $3\frac{1}{7}$? in $5\frac{2}{7}$?

7. How many quarter dollars are there in \$2? \$5 $\frac{1}{4}$? \$3 $\frac{3}{4}$? \$6 $\frac{1}{4}$? \$10? \$75?

8. Change to improper fractions: $3\frac{1}{5}$; $2\frac{1}{6}$; $6\frac{1}{6}$; $5\frac{1}{11}$; $4\frac{7}{12}$; $7\frac{2}{5}$; \$10 $\frac{1}{2}$; \$25 $\frac{5}{8}$.

9. Change to 9ths: $3\frac{1}{9}$; $2\frac{4}{9}$; $7\frac{7}{9}$; $8\frac{4}{9}$; $9\frac{1}{9}$; $11\frac{1}{9}$.

10. How many 10ths are there in 1? $1\frac{3}{10}$? $1\frac{6}{10}$? 2? $5\frac{4}{10}$? 100? 200? 500?



1. Twelve twelfths.

$$1 = \frac{12}{12}.$$



2. One

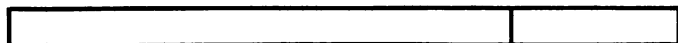
one half

$$1\frac{1}{2} = \frac{3}{2}.$$



3. Twelve eighths.

$$\frac{12}{8} = \frac{3}{2}.$$



4. One

one third

$$1\frac{1}{3} = \frac{4}{3}.$$



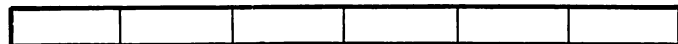
5. Four thirds.

$$\frac{4}{3}.$$



6. Twelve ninths.

$$\frac{12}{9} = \frac{4}{3}.$$



7. Six sixths.

$$\frac{6}{6} = 1.$$

Six fifths.

$$\frac{6}{5} = 1\frac{1}{5}.$$

Six fourths.

$$\frac{6}{4} = 1\frac{3}{4} = 1\frac{1}{2}.$$

Six thirds.

$$\frac{6}{3} = 2.$$

Six halves.

$$\frac{6}{2} = 3.$$

Six.

$$6 = 6 \text{ ones.}$$

8. Show that $1\frac{2}{10} = 1\frac{2}{10} = 1\frac{1}{5}$.

A mixed number is a unit, or several or many units, with a fraction of the unit added.

9. Draw an inch and seven eighths; and $1\frac{7}{8}$ inches.

RATIO AND FRACTION DRAWINGS

Notice the fraction drawings on pages 85, 86. They are made with exactness. But in showing the principles of fractions, outside of books, it is not necessary to make drawings with equal exactness. Freehand drawings, made without rulers or straight edges, show the truth sufficiently well.

Circles may be made by tracing around nickel or silver coins or spools. Other methods are explained on page 118.

1. Show the equivalence of $\frac{3}{8}$, $\frac{6}{16}$, and $\frac{9}{24}$ by drawings.
2. Show the ratios of 3, 5, 8, 10 to each other by drawings.
3. Show by drawings the ratios $\frac{3}{8}$, $\frac{3}{8}$, $\frac{3}{10}$.
4. Show also the ratios $\frac{5}{8}$, $\frac{5}{8}$, $\frac{5}{10}$.
5. Show the equivalence of $\frac{4}{5}$ and $\frac{8}{10}$.
6. Show by a drawing the ratio of a gallon to a quart.
7. Show also the ratio of a peck to a quart.
8. Show the ratio of a foot to a yard.
9. Show the ratio of $2\frac{1}{2}$ to $\frac{1}{4}$.
10. Show the ratio of $\frac{9}{8}$ to 1. •

GENERAL REVIEW

WRITE

11. Add : $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{10}$; $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{9}$, and $\frac{1}{12}$.
12. Subtract : $\frac{7}{12}$ from $\frac{11}{15}$; $\frac{5}{6}$ from $\frac{24}{5}$.
13. Add : \$2 $\frac{1}{3}$, \$5 $\frac{2}{3}$, and \$8 $\frac{3}{4}$.
14. Reduce to mixed numbers : $\frac{30}{9}$, $\frac{8}{3}$, $\frac{43}{7}$, and $\frac{22}{10}$.
15. Reduce to improper fractions : $2\frac{1}{6}$, $8\frac{3}{8}$, and $1\frac{83}{100}$.

ADDITION OF FRACTIONS

When fractions have the same denominators, we can add them by adding their numerators.

$$\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{5}{4} = \frac{11}{4} \qquad \frac{11}{4} = 2\frac{3}{4}$$

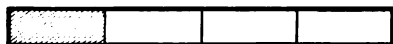
We do not add the denominators because the denominators tell us only the number of fractional parts in the units; the size of these fractional parts remains the same.

Add : $\frac{2}{9}, \frac{3}{9}, \frac{7}{9}$, and $\frac{1}{9}$. $\frac{2}{9} + \frac{3}{9} + \frac{7}{9} + \frac{1}{9} = \frac{13}{9} = 1\frac{4}{9}$

But we cannot at once add fractions with different denominators.

By these forms it is easy to see why it is impossible at once to add fractions that represent equal units divided into unequal parts.

$$\frac{2}{5} + \frac{1}{4} + \frac{1}{3} = ?$$



If we divide these forms into more equal parts, that is, into smaller fractional parts, then we can add these fractions of the forms together. But into how small parts shall we divide them? The *least common multiple* tells us.

5, 4, and 3 have no common factors. $5 \times 4 \times 3 = 60$.

We must divide these forms into 60 equal parts.

$$\frac{2}{5} = \text{how many sixtieths?} \quad 60 \div 5 = 12 \quad 12 \times 2 = 24 \quad \frac{2}{5} = \frac{24}{60}$$

$$\frac{1}{4} = \text{how many sixtieths?} \quad 60 \div 4 = 15 \quad 15 \times 1 = 15 \quad \frac{1}{4} = \frac{15}{60}$$

$$\frac{1}{3} = \text{how many sixtieths?} \quad 60 \div 3 = 20 \quad 20 \times 1 = 20 \quad \frac{1}{3} = \frac{20}{60}$$

$$\frac{24}{60} + \frac{15}{60} + \frac{20}{60} = \frac{59}{60}$$

Since 59 and 60 have no common factor, this fraction is at its lowest terms. No smaller denominator would be large enough to help us to add these fractions together.

REDUCING FRACTIONS

Sometimes we have such fractions as $\frac{2}{3}$, $\frac{6}{12}$, $\frac{4}{20}$, and prefer to have them changed to smaller numerators and denominators.

We know that $\frac{1}{4} = \frac{2}{8}$. Dividing both the numerator and the denominator of $\frac{2}{8}$ by 2 does not change its real value, because, though we make each equal part as shown by the denominator twice as large, we take only half as many equal parts. Whenever we change the sizes of denominator and numerator in such a way as not to change their ratio to each other, the value of the fraction is unchanged.

1. Reduce $\frac{120}{180}$ to its lowest terms.

$$\begin{array}{l} 10 \overline{) 120} = \frac{12}{18} \quad 2 \overline{) 12} = \frac{6}{9} \quad 3 \overline{) 6} = \frac{2}{3} \end{array}$$

We divide each of the terms of the fraction $\frac{120}{180}$ by 10, 2, and 3, factors common to both numerator and denominator, and obtain $\frac{2}{3}$. The numerator and denominator 2 and 3 are prime to each other. Hence the fraction $\frac{2}{3}$ is in its lowest terms.

Another way :

$$\begin{array}{r} 10 \overline{) 120} \quad 180 \quad \text{greatest} \\ 3 \overline{) 12} \quad 18 \quad \text{common divisor} \\ 2 \overline{) 4} \quad 6 \\ \hline 2 \quad 3 \\ 10 \times 2 \times 3 = 60 \end{array}$$

$$\begin{array}{l} 60 \overline{) 120} = \frac{2}{3} \\ 60 \overline{) 180} \end{array}$$

Reduce to lowest terms these fractions :

2. $\frac{48}{96}$.

3. $\frac{75}{100}$.

4. $\frac{72}{100}$.

5. $\frac{18}{27}$.

6. $\frac{15}{25}$.

7. $\frac{21}{28}$.

8. $\frac{8}{36}$.

9. $\frac{12}{96}$.

10. $\frac{96}{144}$.

11. $\frac{72}{96}$.

12. $\frac{48}{54}$.

13. $\frac{63}{72}$.

14. $\frac{36}{108}$.

15. $\frac{24}{56}$.

16. $\frac{39}{69}$.

17. $\frac{24}{144}$.

SUBTRACTION OF FRACTIONS

1. From \$5 take
- $\frac{4}{5}$
- of a dollar.

$$\$5 = \$4\frac{5}{5}$$

$$4\frac{5}{5} - \frac{4}{5} = \$4\frac{1}{5}$$

2. From
- $1\frac{3}{4}$
- take
- $1\frac{1}{9}$
- .

$$1\frac{3}{4} = \frac{7}{4}$$

$$1\frac{1}{9} = \frac{10}{9}$$

$$\frac{7}{4} - \frac{10}{9} = \frac{63}{36} - \frac{40}{36} = \frac{23}{36}$$

In this example we reduce the mixed numbers to improper fractions, and these fractions to a common denominator. Finding the difference between the numerators, we write it over the common denominator, and obtain the required difference, $\frac{23}{36}$.

3. From
- $7\frac{1}{3}$
- take
- $6\frac{7}{8}$
- .

$$7\frac{1}{3} = 6 + \frac{2}{3} + \frac{1}{3} = 6\frac{4}{3}$$

$$6\frac{4}{3} - 6\frac{7}{8} = 6\frac{32}{24} - 6\frac{21}{24}$$

$$\begin{array}{r} 6\frac{32}{24} \\ - 6\frac{21}{24} \\ \hline 1\frac{11}{24} \end{array}$$

4. Take
- $2\frac{3}{8}$
- from
- $4\frac{1}{4}$
- .

$$4\frac{1}{4} = 4\frac{2}{8}$$

$$4\frac{2}{8} - 2\frac{3}{8} = 1\frac{7}{8}$$

We reduce the fractions to a common denominator. Then, as we cannot take $\frac{3}{8}$ from $\frac{2}{8}$, we add 1, taken from 4, or $\frac{8}{8}$ to the $\frac{2}{8}$, making $1\frac{10}{8}$.

Another way is to reduce both mixed numbers to improper fractions.

$$4\frac{2}{8} = \frac{34}{8}$$

$$2\frac{3}{8} = \frac{19}{8}$$

$$\frac{34}{8} - \frac{19}{8} = \frac{15}{8}$$

$$\frac{15}{8} = 1\frac{7}{8}$$

Find the answers to these questions:

5. $2\frac{3}{8} - 1\frac{5}{8} = ?$

6. $22\frac{1}{3} - 21\frac{5}{9} = ?$

7. $5\frac{3}{14} - 3\frac{6}{7} = ?$

8. $20\frac{11}{28} - 4\frac{11}{21} = ?$

9. $5\frac{3}{14} - 3\frac{6}{7} = ?$

10. $20\frac{11}{28} - 4\frac{11}{21} = ?$

11. $5 - 3\frac{8}{13} = ?$

12. $200 - 17\frac{7}{9} = ?$

13. $27\frac{1}{3} - 26\frac{2}{7} = ?$

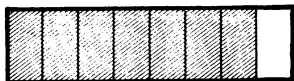
14. $8\frac{7}{24} - 7\frac{11}{12} = ?$

15. $11\frac{7}{33} - 2\frac{1}{2} = ?$

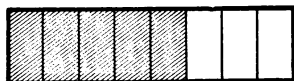
16. $1\frac{12}{50} - \frac{99}{100} = ?$

SUBTRACTION OF FRACTIONS

SAME DENOMINATOR

From $\frac{7}{8}$ take $\frac{5}{8}$. $\frac{7}{8}$ 

$$\frac{7}{8} - \frac{5}{8} = \frac{2}{8}$$

 $\frac{5}{8}$ 

Since both fractions are eighths, we need to find only the difference, 2, between the numerators, and to write it over the common denominator, 8.

1. From $\frac{7}{11}$ take $\frac{2}{11}$.

2. From $\frac{11}{19}$ take $\frac{7}{19}$.

3. From $\frac{27}{37}$ take $\frac{4}{37}$.

4. From $\frac{167}{711}$ take $\frac{19}{711}$.

5. From $\frac{7}{20}$ take $\frac{5}{20}$.

6. From $\frac{97}{100}$ take $\frac{17}{100}$.

WRITE

DIFFERENT DENOMINATORS

From $\frac{17}{18}$ take $\frac{7}{12}$.

Here the denominators are different, and we must first reduce the fractions to a common denominator.

$$18 = 3 \times 3 \times 2$$

$$12 = 3 \times 2 \times 2$$

Hence, the least common denominator is $3 \times 3 \times 2 \times 2 = 36$.

$$\frac{17}{18} = \frac{34}{36}$$

$$\frac{7}{12} = \frac{21}{36}$$

$$\frac{34}{36} - \frac{21}{36} = \frac{13}{36}$$

7. From $\frac{3}{4}$ take $\frac{1}{2}$.

8. From $\frac{2}{7}$ take $\frac{4}{21}$.

9. $\frac{3}{7} - \frac{4}{10} = ?$

Answer the following questions:

10. $\frac{5}{8} - \frac{2}{5} = ?$

11. $\frac{7}{11} - \frac{2}{7} = ?$

12. $\frac{2}{9} - \frac{1}{7} = ?$

13. $\frac{5}{13} - \frac{4}{11} = ?$

14. $\frac{17}{20} - \frac{4}{5} = ?$

15. $\frac{23}{24} - \frac{43}{48} = ?$

16. $\frac{3}{5} - \frac{1}{3} = ?$

17. $\frac{1}{2} - \frac{1}{5} = ?$

18. $\frac{11}{12} - \frac{19}{24} = ?$

19. $\frac{15}{16} - \frac{27}{32} = ?$

20. $\frac{9}{10} - \frac{17}{20} = ?$

21. $\frac{21}{40} - \frac{19}{80} = ?$

FRACTIONS

Like fractions indicate the same number of equal parts in the unit. Like fractions have the same denominator.

Unlike fractions have different denominators, which indicate different numbers of equal parts in the unit.

1. Reduce to improper fractions these mixed numbers :

$$7\frac{1}{2}, 15\frac{5}{8}, 4\frac{7}{11}, 102\frac{5}{9}, 5\frac{9}{10}, 19\frac{3}{4}, 9\frac{19}{21}, 236\frac{4}{19},$$

$$10\frac{11}{12}, 8\frac{8}{10}, 19\frac{3}{4}, 19\frac{19}{20}, 28\frac{7}{10}, 10\frac{1}{19}, 215\frac{5}{14}, 12\frac{3}{5},$$

$$10\frac{11}{12}, 15\frac{4}{27}, 84\frac{3}{26}, 20\frac{5}{11}, 20\frac{9}{35}, 25\frac{4}{6}, 100\frac{7}{8}, 132\frac{4}{25}.$$

2. Reduce these improper fractions to mixed numbers :

$$\frac{11}{4}, \frac{15}{3}, \frac{19}{5}, \frac{21}{6}, \frac{35}{7}, \frac{41}{6}, \frac{54}{9}, \frac{63}{7},$$

$$\frac{839}{60}, \frac{31}{10}, \frac{72}{11}, \frac{95}{12}, \frac{108}{9}, \frac{121}{18}, \frac{111}{24}, \frac{216}{19},$$

$$\frac{718}{44}, \frac{908}{60}, \frac{910}{57}, \frac{311}{47}, \frac{926}{58}, \frac{1011}{74}, \frac{2016}{98}, \frac{26}{7},$$

$$\frac{92}{9}, \frac{94}{11}, \frac{2000}{73}, \frac{65}{14}, \frac{236}{17}, \frac{426}{13}, \frac{863}{12}, \frac{101}{17},$$

$$\frac{100}{31}, \frac{112}{7}, \frac{117}{16}, \frac{289}{17}, \frac{200}{33}, \frac{110}{12}, \frac{201}{8}, \frac{50}{7}.$$

We have practiced **factoring** and **cancellation**, have found **greatest common divisors** and **least common multiples**, and are studying **improper fractions**, and **mixed numbers** partly because we need to know these processes so as to add, to subtract, to multiply, and to divide fractions.

Tell the meaning of each of the expressions in **boldface** type.

These processes are not so important in themselves as are addition, subtraction, multiplication, and division of whole numbers, but without knowing them thoroughly we can never succeed in the later and higher subjects of Arithmetic.

ADDITION OF MIXED NUMBERS

Add: $31\frac{1}{5}$, $3\frac{5}{6}$, and $15\frac{3}{10}$.

$$\begin{array}{r} 31 \\ 3 \end{array}$$

$$\frac{2}{5} = \frac{12}{30}.$$

$$\frac{5}{6} = \frac{25}{30}.$$

$$\frac{3}{10} = \frac{9}{30}.$$

$$\begin{array}{r} 15 \\ 49 \end{array}$$

$$\frac{2}{5} + \frac{5}{6} + \frac{3}{10} = \frac{12}{30} + \frac{25}{30} + \frac{9}{30} = \frac{46}{30} = \frac{23}{15} = 1\frac{8}{15}.$$

$$\begin{array}{r} 1\frac{8}{15} \\ 50\frac{8}{15} \end{array}$$

Add:

- | | |
|---|--|
| 1. $5\frac{1}{9}$, $6\frac{8}{9}$, 7. | 2. $3\frac{1}{5}$, $2\frac{4}{5}$. |
| 3. $4\frac{1}{3}$, $5\frac{2}{5}$, $1\frac{4}{15}$. | 4. $2\frac{2}{3}$, $4\frac{5}{6}$. |
| 5. $2\frac{1}{5}$, $2\frac{1}{15}$, $\frac{1}{9}$, 1. | 6. $6\frac{1}{4}$, $5\frac{3}{8}$, $7\frac{7}{12}$, 11. |
| 7. $1\frac{1}{2}$, $2\frac{3}{4}$, $3\frac{7}{8}$, $4\frac{7}{16}$. | 8. $5\frac{5}{6}$, $4\frac{5}{9}$, $3\frac{1}{18}$, $\frac{5}{12}$. |
| 9. $12\frac{1}{11}$, $1\frac{3}{22}$, $7\frac{5}{33}$. | 10. $\frac{12}{3}$, $3\frac{1}{5}$, $\frac{5}{12}$, $\frac{7}{20}$. |
| 11. $1\frac{3}{14}$, $\frac{60}{49}$, $1\frac{1}{7}$, 2. | 12. $11\frac{1}{18}$, $2\frac{7}{27}$, $\frac{5}{54}$, $\frac{2}{9}$. |
| 13. $3\frac{2}{7}$, $5\frac{8}{3}$, $3\frac{11}{21}$, 2. | 14. $4\frac{1}{7}$, $4\frac{1}{14}$, $4\frac{1}{28}$, $\frac{1}{49}$. |
| 15. $2\frac{7}{5}$, $4\frac{8}{25}$, $1\frac{01}{15}$. | 16. $\frac{41}{22}$, $1\frac{5}{33}$, $1\frac{5}{6}$. |
| 17. $\frac{99}{40}$, $7\frac{9}{25}$, $8\frac{7}{50}$. | 18. $1\frac{000}{24}$, $1\frac{00}{8}$, $1\frac{0}{3}$. |
| 19. $\frac{63}{7}$, $\frac{45}{14}$, $1\frac{00}{28}$. | 20. $\frac{300}{21}$, $\frac{400}{49}$, $\frac{5}{8}$. |
| 21. $2\frac{3}{5}$, $5\frac{1}{10}$, $2\frac{11}{10}$. | 22. $1\frac{1}{4}$, $4\frac{7}{12}$, $3\frac{1}{16}$, $2\frac{7}{4}$. |
| 23. 2, $3\frac{4}{11}$, $\frac{231}{65}$, $\frac{491}{33}$. | 24. $1\frac{1}{9}$, $1\frac{1}{30}$, $1\frac{1}{15}$, $1\frac{1}{45}$. |
| 25. 4, $5\frac{1}{9}$, $\frac{271}{12}$, $4\frac{8}{5}$. | 26. $2\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{7}{8}$. |
| 27. $2\frac{5}{7} + \frac{3}{7} + 4\frac{9}{14} + 8$. | 28. $3\frac{3}{7} + 4\frac{11}{14} + 5\frac{23}{8}$. |
| 29. $3\frac{1}{5} + 2\frac{3}{10} + 5\frac{11}{15} + 6\frac{7}{20}$. | 30. $\frac{7}{18} + 2\frac{1}{4} + 8 + 8\frac{5}{8}$. |
| 31. $13\frac{1}{9} + 5\frac{3}{10} + 1\frac{1}{5}$. | 32. $\frac{1}{3} + 2\frac{1}{5} + 7\frac{1}{9} + 4\frac{5}{12}$. |
| 33. $3\frac{2}{3} + 4\frac{2}{5} + \frac{1}{15}$. | 34. $4\frac{1}{9} + 7\frac{1}{5} + 3\frac{11}{45}$. |
| 35. $2\frac{1}{8} + 7\frac{1}{5} + 2\frac{3}{10}$. | 36. $\frac{2}{3} + \frac{8}{9} + 2\frac{1}{2} + 3$. |

REVIEW

1. If \$34,051 be divided into 12 equal parts, how many dollars will there be in one of the parts?

2. A box contained 1128 eggs. How many dozen eggs were there in the box?

3. A grocer bought 1416 eggs at 9¢ a dozen. How much did they cost him?

4. If 7 dozen eggs cost 168¢, what was the price a dozen? What was the cost of each egg?

5. The wages of 13 men on equal pay for one week amounted to \$97.50. How much did each earn a day?

6. The expense of building a bridge was \$8743, and that of opening a road was \$2163. The total expenses were borne equally by seven towns. How much did each town pay?

7. A man receives a salary of \$1200 a year. Out of this he saves \$212 each year. How much does he spend a week on the average?

8. A contractor requires one million bricks. He has on hand 559,941 bricks. How many loads of 487 bricks each does he need to make up the full number?

9. A bushel of wheat weighs 60 pounds, and a bushel of oats weighs 34 pounds. How many bushels of oats will weigh as much as 187 bushels of wheat?

10. There are how many feet in a mile? How many steps, each 2 feet long, would you take in walking a mile?

11. Seven men have each an equal interest in a farm of 107 acres. They sell it at \$56 an acre. How many dollars should each receive?

QUESTIONS

Many practical problems that men have to solve can be solved only when we understand **equivalence of fractions, reducing fractions to lowest terms, finding least common multiple, and addition and subtraction of fractions.** Be ready to tell orally the meaning of each expression in **bold-face type.**

1. Mr. Smith owns $\frac{3}{8}$ of a section of land, Mr. Jones owns $\frac{5}{16}$ of the section, and Mr. Brown owns the remainder of it. What fraction of the section does Mr. Brown own?

2. A section of land contains 640 acres. How many acres has Mr. Smith in his part of the section? How many has Mr. Jones in his?

3. An acre contains how many square feet? How many square feet are there in $\frac{1}{4}$ of an acre? $\frac{3}{8}$? $1\frac{1}{8}$?

4. A lot contains a half acre of land. It is four times as large as the next lot of land. How many square feet does the second lot contain?

5. A man expended $\frac{1}{3}$ of his salary for board, $\frac{1}{5}$ of it for clothing, $\frac{1}{8}$ of it for books, and $\frac{3}{10}$ of it for other purposes. What part of his salary had he then left?

6. What number must be added to the sum of $1\frac{7}{10}$ and $2\frac{2}{3}$ to make $26\frac{1}{4}$?

7. Charles paid a debt of $\$7\frac{3}{5}$ out of a purse containing $\$9\frac{1}{12}$. How many dollars remained?

8. A person who had $13\frac{5}{8}$ yards of cloth sold $7\frac{3}{4}$ yards. How many yards remained?

9. A milkman sold $33\frac{3}{4}$ gal. of milk from $36\frac{1}{2}$ gal. How many gallons remained? How many quarts?

REVIEW

1. A, B, C, and D built a mill. A invested \$7418; B invested \$9475; C, \$8643; and D, \$7464. How much did the mill cost the builders?

2. In a cattle train there were two cars with 17 head of cattle in each, three cars with 19 head in each, one car with 22 head, and two cars with 21 head in each. How many cattle were there in all the cars?

3. A farmer has 27 A. of land under wheat, 15 A. under oats, 14 A. in meadow, 19 A. in pasture, 9 A. under peas, 6 A. in potatoes, 7 A. in turnips, 19 A. under corn, 5 A. in orchard, 3 A. under garden vegetables, and 20 A. of woods. How many acres are there in his farm?

4. A man bought three farms. For the first farm he paid \$4793 for the land, \$479 for the crop, and \$698 for the live stock; for the second he paid \$5986 for the land, \$973 for the crop, and \$546 for the live stock; for the third he paid \$8678 for the land, \$1094 for the crop, and \$783 for the live stock. What was the total amount paid?

5. Franklin was born in 1706 and died in 1790. Washington was born in 1732 and died in 1799. How old was each when he died?

6. Our Declaration of Independence was adopted in 1776. The landing of the Pilgrims took place 156 years before the Declaration was adopted, and it is said that gunpowder was invented in Europe 200 years before the landing of the Pilgrims. Give the supposed year of the invention of gunpowder in Europe.

7. France contains 204,092 square miles. Texas contains 265,780 square miles. How many more square miles are there in Texas than in France?

LIQUID MEASURE

Liquid measure is used in measuring all kinds of liquids.

TABLE

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

In measuring the capacity of reservoirs, tanks, cisterns, etc., $31\frac{1}{2}$ gal. make a barrel (bbl.), and 63 gal. a hogshead (hhd.), but in commerce the barrel and the hogshead vary in capacity. The standard unit of liquid measure is the gallon. It contains 231 cu. in.

WRITE

1. How many quarts are there in 2 gal.? 3 gal.? 4 gal.?
2. How many pints are there in 2 qt.? 3 qt.? 5 qt.?
3. How many gills are there in 2 pt.? 3 pt.? 6 pt.?
4. How many quarts are there in 16 pt.? 24 pt.? 21 pt.?
5. How many gallons are there in 28 qt.? 32 qt.? 25 qt.? 36 qt.? 40 pt.?
6. How many quarts are there in $\frac{1}{2}$ gal.? $\frac{7}{8}$ gal.?
7. What is the value of a can of milk containing $66\frac{2}{3}$ pt. at 6¢ a quart?
8. A grocer sold in one month 8 bbl. of oil at 50¢ a gallon. What was the total amount of money he received?
9. He bought 3 hhd. of molasses at \$24 a hhd. and sold the molasses at 50¢ a gal. What did he gain?
10. A reservoir holds 30 000 000 bbl. of water. At 10 gal. a day each how long would this last 1000 people?

DRY MEASURE

Dry measure is used in measuring grain, fruits, and vegetables.

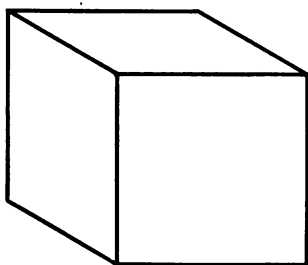
TABLE

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
$2\frac{1}{2}$ bushels	= 1 barrel (bbl.)

WRITE

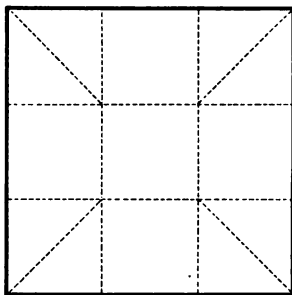
- How many pecks are there in 2 bu.? 4 bu.? 5 bu.?
- How many quarts are there in 2 pk.? 3 pk.? 6 pk.?
- How many pints are there in 2 qt.? 7 qt.? 9 qt.?
- How many pints are there in 1 bu.? 2 bu.?
- How many quarts are there in $\frac{1}{2}$ bu.? $\frac{1}{4}$ bu.? $\frac{3}{4}$ bu.?
- How many pecks are there in $\frac{1}{4}$ bu.? $\frac{1}{2}$ bu.? $1\frac{1}{4}$ bu.?
- How many pecks are there in 23 qt.? 41 qt.?
- How many bushels are there in 8 pk.? 17 pk.?
27 pk.? 32 pk.?
- John picked 64 pk. of strawberries at 3¢ a quart. His employer sold them at 8¢ a quart. How much did John receive? What did his employer gain, if his other expenses were not counted?
- A housekeeper bought 2 bu. of potatoes and used them at the rate of 3 pt. a day. How long did they last?
- An apple orchard bore 100 bu. of apples. The farmer sold $\frac{1}{2}$ of them at \$1 a barrel. How many dollars did he receive for these apples?
- A family used 2 bbl. of apples in one month. How many pt. was this per day?

MEASURES OF CAPACITY



Cubic inch

$$1\frac{1}{2} \times 1\frac{1}{2}$$

 $1\frac{1}{2}$ in. $\times$ $1\frac{1}{2}$ in. ($\frac{1}{2}$ size)

4 gills make 1 pint.

1. Take a liquid gill measure and find how many cubic inches it contains.

A cubic inch may be made out of cardboard or heavy paper. Fold at the lines dotted. This will hold water, but dry sand may be used.

A cardboard box of the volume given by sides 3 in. $\times$ 7 in. $\times$ 11 in. will be found to contain almost exactly 1 gallon.

2. Make the box of paper 3 in. $\times$ 3 in.

3. Find the number of cubic inches in a dry quart.

4. Find the contents in cubic inches of a drinking glass. Compare this with a pint.

5. What is the capacity in cubic feet of a distributing city reservoir 20 ft. $\times$ 500 ft. $\times$ 3000 ft. ?

6. What is the volume of a square prism 3 in. by 3 in. by 2 ft. ? of a prism 5 ft. by 2 ft. by 2 ft. ?

AVOIRDUPOIS WEIGHT

Avoirdupois weight is used for weighing all articles except gold, silver, jewels, and medicines when compounded.

TABLE

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight	} = 1 ton (T.)
2000 pounds	

WRITE

1. How many pounds are there in 2 T.? 3 T.? 4 T.? 10 T.?

2. How many pounds are there in 2 cwt.? 3 cwt.? 5 cwt.? 8 cwt.?

3. How many ounces are there in 2 lb.? 3 lb.? 5 lb.?

4. How many tons are there in 6000 lb.? in 8000 lb.? in 20,000 lb.?

5. How many ounces are there in $\frac{1}{2}$ lb.? in $1\frac{1}{2}$ lb.?

6. How many ounces are there in 1 cwt.?

7. John bought 8 oz. of soda. What part of a pound was that?

8. He bought also $\frac{1}{2}$ ton of coal. How many pounds was that? If his mother used two buckets of coal a day, and each bucket weighed 20 pounds, how many days did the coal last?

9. A hay wagon weighed 1400 pounds, and the hay on it weighed 3600 pounds. The two horses weighed each 1100 pounds, while two men on the wagon weighed each 150 pounds. Was it safe to cross a bridge built to support 3 tons?

A "long ton" used for coal at the mines and for merchandise at the United States Custom Houses weighs 2240 pounds.

MEASURE OF TIME

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 year (yr.)
366 days	= 1 leap year (yr.)
100 years	= 1 century (C.)

Leap year comes every four years, when February gains a day.

The year is divided into 12 calendar months:

January (Jan.)	. 31 da.	July	31 da.
February (Feb.)	. 28 da.	August (Aug.)	. 31 da.
March (Mar.)	. . 31 da.	September (Sept.)	30 da.
April (Apr.)	. . 30 da.	October (Oct.)	. 31 da.
May	31 da.	November (Nov.)	. 30 da.
June	30 da.	December (Dec.)	. 31 da.

Thirty days hath September,
 April, June, and November,
 All the rest have thirty-one,
 Excepting February alone.
 Twenty-eight are all its store
 Till leap year gives it one day more.

Until the year 2400 every
 year we can divide by 4 will
 be leap year. We usually call
 thirty days a month unless
 we know the exact month in
 question.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

This calendar is true for any
 month when the first day of the
 month falls on Sunday and when
 the month has 31 days. This
 calendar represents December,
 1901, and March, 1903. If the
 31st day were omitted, it would
 represent June, 1902, and No-
 vember, 1903, also.

COUNTING

In counting many kinds of articles we use these measures :

- 12 things = 1 dozen (doz.)
- 12 dozen = 1 gross (gr.)
- 12 gross = 1 great gross (G. gr.)
- 20 things = 1 score (sc.)

In buying and selling paper we use these measures :

- 24 sheets = 1 quire (qr.)
- 20 quires = 1 ream (R.)
- 2 reams = 1 bundle (bdl.)
- 5 bundles = 1 bale.

Reams of 500 sheets are now very common.

WRITE

1. How many lead pencils are there in $\frac{1}{4}$ gross ?
2. How many sheets of paper are there in a ream ?
3. How many quires are there in 2 R. ? in 5 R. ?
4. How many score are there in 100 ?
5. Name three kinds of articles sold by the dozen.
6. A school used 1 great gross of pens every term. How many single pens were used in three terms ?
7. In a business office 4 reams of stationery were used in one month. How many sheets were used ?
8. A daily newspaper in a great city used 120 bales of paper a day. How many sheets were used ?
9. In her correspondence Mrs. Somers used in one year 25 quires of stationery. How many sheets was this ?
10. How many things make 4 G. gr. ?

BILLS

A bill of goods is a written account of articles sold or of services rendered, with dates, place, prices, and names of creditor and debtor; that is, of the person to whom the money is due and of the person who owes the money.

New York, Aug. 1, 1901.

William Estabrook,

1291 Fifth Ave., New York.

Bought of Norton & Wright.

<i>July</i>	<i>5</i>	<i>½ yd. Flannel</i>	<i>.50</i>	<i>25</i>		
		<i>pr. Rubbers</i>		<i>70</i>		
		<i>pr. Shoes</i>		<i>2 50</i>		
		<i>Tie</i>		<i>1</i>		
		<i>Basket</i>		<i>98</i>		
	<i>10</i>	<i>30 yd. Carpet</i>	<i>1.25</i>	<i>37 50</i>		
	<i>15</i>	<i>2 Shirts</i>	<i>.98 69</i>	<i>1 67</i>		
	<i>24</i>	<i>10 yd. Silk</i>	<i>2.35</i>	<i>23 50</i>	<i>68</i>	<i>10</i>
	<i>10</i>	<i>By Cash</i>			<i>30</i>	
		<i>Balance due</i>			<i>38</i>	<i>10</i>
		<i>Received payment</i>				
		<i>Norton & Wright.</i>				
		<i>Aug 5, 1901.</i>				

BILLS

Make out the following bills :

1. Mrs. C. Allen bought of Langdon & Co., Brooklyn, Oct. 25, 1900, 14 yd. print at 13¢ per yd ; Nov. 3, 33 yd. cotton at 14¢ ; Nov. 6, 17 yd. tweed at \$1.18 ; Nov. 12, 16 yd. silk at \$1.87 ; 9 yd. lining at 13¢ ; 3 doz. buttons at 23¢ ; Nov. 14, 9 yd. jersey cloth at 45¢ ; 2 yd. plush at \$1.95 ; 3 yd. lining at 18¢ ; 2 doz. buttons at 15¢.

2. Mrs. M. A. Taylor bought of Race & Co., Rochester, N. Y., Sept. 27, 1900, 9 lb. roast beef at 12¢ ; Nov. 21, 7 lb. lamb at 13¢ ; Nov. 23, 8 lb. beef at 8¢ ; Nov. 25, 3 lb. steak at 14¢ ; Nov. 26, 13 lb. lamb at 13¢ ; Nov. 28, 5 lb. corned beef at 9¢, and 2 geese at 65¢ each ; Dec. 1, 3 chickens at 55¢, and 9 lb. sausages at 12¢.

3. Walter Hays, Omaha, sold to Harvey Richards, on Aug. 20, 1900, 14 cords maple at \$8.50, 4 cords soft wood at \$3.25, and 7 cords beech at \$7.25.

4. George Pinkham bought of Horace Shaw, Kansas City, Sept. 11, 1900, 700 lb. of rye at \$2.75 per cwt. ; 400 lb. oatmeal at \$2.25 ; 300 lb. cornmeal at \$2.25 ; and 200 lb. buckwheat flour at \$2.50. On Sept. 28, 1900, George Pinkham paid \$25 on the above account.

5. C. L. Cook bought of Williams & Co., Louisville, April 16, 1900, 100 ft. $\frac{3}{4}$ in. rubber hose at 20¢ per ft. ; 2 pr. $\frac{3}{4}$ in. couplings and fittings at 50¢ per pr. ; 1 hose pipe, at \$1.25 ; 3 step-ladders at \$1.50.

6. Williams & Jones, New York, sold to R. M. Marston, Dec. 5, 1899, 19 yd. calico at 7¢ ; 17 yd. linen at 47¢ ; 16 yd. lining at 9¢ ; Dec. 21, 8 yd. flannel at 48¢ ; 23 yd. braid at 3¢ ; Dec. 26, 7 pr. stockings at 25¢ ; and 3 pr. gloves at 65¢. Paid in full, Jan. 2, 1900.

CANCELLATION

Sometimes we can simplify division by removing factors common to both dividend and divisor.

1. Divide 450 by 150.

$\begin{array}{r} 45 \\ 450 \end{array} \div \begin{array}{r} 15 \\ 150 \end{array}$ We divide both dividend and divisor by the common factor, 10.

$\begin{array}{r} 9 \\ 45 \end{array} \div \begin{array}{r} 3 \\ 15 \end{array}$ We divide by 5.

$\begin{array}{r} 3 \\ 9 \end{array} \div \begin{array}{r} 1 \\ 3 \end{array}$ We divide by 3.

$3 \div 1 = 3$, the quotient of $450 \div 150$

Write in this way :

2. Explain this :

$\begin{array}{r} 3 \\ 9 \\ 45 \\ 450 \end{array} \div \begin{array}{r} 1 \\ 3 \\ 15 \\ 150 \end{array}$ $3 \div 1 = 3$

$\begin{array}{r} 8 \\ 40 \\ 200 \\ 1000 \end{array} \div \begin{array}{r} 1 \\ 5 \\ 25 \\ 125 \end{array}$ $8 \div 1 = 8$

Removing common factors in division is **cancellation**.

3. What is the quotient of $45 \times 12 \times 16 \times 15$ divided by $9 \times 24 \times 5 \times 10$?

$$\frac{\begin{array}{r} 1 \\ 3 \\ 15 \\ 45 \end{array} \times \begin{array}{r} 1 \\ 2 \\ 10 \\ 24 \end{array} \times \begin{array}{r} 1 \\ 3 \\ 15 \\ 16 \end{array} \times \begin{array}{r} 1 \\ 3 \\ 15 \\ 12 \end{array}}{\begin{array}{r} 3 \\ 9 \\ 24 \\ 8 \end{array} \times \begin{array}{r} 1 \\ 2 \\ 10 \\ 5 \end{array} \times \begin{array}{r} 1 \\ 3 \\ 15 \\ 5 \end{array} \times \begin{array}{r} 1 \\ 3 \\ 15 \\ 1 \end{array}} = \frac{1 \times 1 \times 1 \times 12}{1 \times 1 \times 1 \times 1} = \frac{12}{1} = 12.$$

4. Divide 22×15 by 11×5 .

5. Divide $24 \times 16 \times 25 \times 20$ by $10 \times 4 \times 6 \times 5 \times 40$.

Arrange the numbers in order of their size.

Be very careful to cancel only once each in dividend and divisor for any one common factor.

CANCELLATION

Divide: 1. 25×11 by 5×11 .

2. 56×24 by 7×48 .

3. $4 \times 6 \times 12$ by $3 \times 8 \times 4$.

4. $3 \times 50 \times 8$ by $4 \times 5 \times 2 \times 10$.

5. $5 \times 16 \times 4$ by 8×20 .

6. $28 \times 33 \times 35$ by $15 \times 11 \times 14$.

$$7. \frac{8 \times 7 \times 81}{27 \times 28}$$

$$8. \frac{112 \times 24 \times 14}{12 \times 28 \times 16}$$

$$9. \frac{36 \times 10 \times 7}{14 \times 5 \times 9}$$

$$10. \frac{35 \times 33 \times 28}{14 \times 15 \times 11}$$

$$11. \frac{21 \times 8 \times 40}{4 \times 7 \times 20}$$

$$12. \frac{17 \times 64 \times 11 \times 90}{30 \times 22 \times 51 \times 8}$$

13. A boy bought 6 dozen eggs at 15¢ a dozen, and raised 4 dozen chickens, which he sold at 10¢ each. What was the ratio of the first cost to the returns?

14. There were 900 private soldiers in a regiment, who received \$12 a month each. The officers received in all \$1080 a month. What was the ratio of the total wages of the privates to the wages of the officers?

15. The salaries of the teachers of 10,000 children a year cost the city of W—— an average of \$24 per child. The salaries of 300 policemen averaged \$800 a year each. What was the ratio of the total cost of free public teaching to the cost of the free public police service?

Do you notice how easy it is to cancel 10's, 100's, and 1000's?

Arithmetic.

Room 14. Grade IV. William Estabrook,
 School No. 5. March 25, 1901.

1. Divide 22×15 by 11×5 .

$$\begin{array}{r} 2 \quad 3 \\ \cancel{22} \times \cancel{15} = \frac{2 \times 3}{1 \times 1} = \frac{6}{1} = 6 \end{array} \quad \text{Answer, 6.}$$

2. Add 10.5, .003, 15.641 and 300; and write the answer in words.

$$\begin{array}{r} 10.5 \\ .003 \\ 15.641 \\ \underline{300} \\ 326.144 \end{array}$$

Answer,

Three hundred twenty-six, and one hundred forty-four thousandths.

3. $x = \frac{\$30 \times 2}{4}$ Find x .

$$\begin{array}{r} 15 \\ x = \frac{\cancel{\$30} \times \cancel{2}}{\cancel{4}} \\ \quad \quad \quad \cancel{2} \\ \quad \quad \quad 1 \end{array} \quad x = \$15$$

Answer, \$15

or

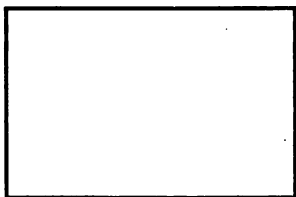
$$\begin{array}{r} x = \frac{\$30 \times 2}{4} \quad 4x = \$30 \times 2 \\ \quad \quad \quad 4x = \$60 \quad x = \$15 \end{array}$$

Answer, \$15

AREAS

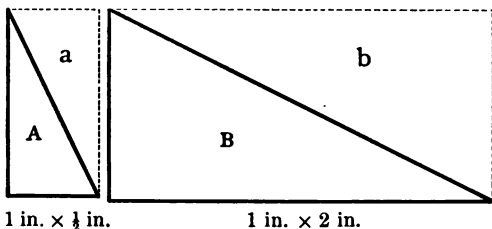
We find the **areas** of rectangles in square measure by multiplying the numbers representing the lengths of the adjoining sides.

1. If the sides of this rectangle were 4 inches and 6 inches, its area would be $4 \text{ sq. in.} \times 6 = 24 \text{ square inches} = 24 \text{ sq. in.}$



2. If a mirror is $3 \text{ ft.} \times 4\frac{1}{2} \text{ ft.}$ in size, its area is $13\frac{1}{2} \text{ sq. ft.}$

We find the areas of right-angled triangles by multiplying the numbers representing the lengths of the sides which make the right angle and dividing the result by two.



The dotted lines show the rectangle that the multiplication of the lengths of the two sides gives us.

3. Measure the triangle A, and find its area upon this scale. Length $1 \text{ in.} = 4 \text{ ft.}$ Width $\frac{1}{2} \text{ in.} = 2 \text{ ft.}$ $4 \text{ sq. ft.} \times 2 = 8 \text{ sq. ft.}$ $8 \text{ sq. ft.} \div 2 = 4 \text{ sq. ft.}$

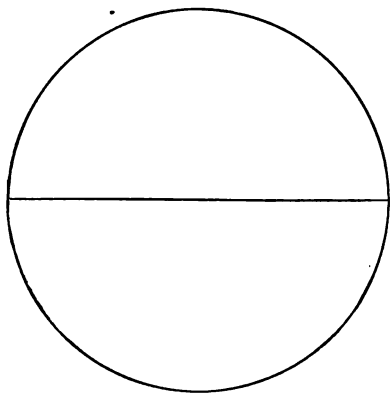
Or by cancellation

$$\frac{4 \text{ sq. ft.} \times \frac{2}{2}}{2} = 4 \text{ sq. ft.}$$

4. By the same scale find the area of the triangle B.

5. Draw various triangles on the blackboard. Measure them. Find their areas, using no fractions smaller than $\frac{1}{4}$.

CIRCUMFERENCES



A circle may be drawn on the blackboard by making a loose knot in a string and setting the knot around the crayon. Then if one holds the free end of the string against the blackboard with one finger of the left hand, and stretches it tight, a circle of any desired diameter may be made. The finger marks the center. With a pin, a pencil, and a piece of cardboard with holes in it for the pin and pencil, circles may be drawn on paper. Circles may also be drawn with dividers or compasses.

The diameter of a circle is twice its radius. A radius is any straight line from the center to the circumference. The string makes the length of the radius of the circle drawn on the blackboard.

The circumference of any circle equals almost exactly three and a seventh times the diameter. We can prove this by drawing circles and comparing their diameters and circumferences.

1. Find the circumference of a circle 2 in. in diameter.

$$2 \text{ in.} \times 3\frac{1}{7} = 6\frac{2}{7} \text{ in.}$$

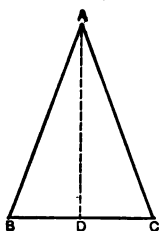
2. Find the circumference of a circle 4 yd. in diameter.

$$4 \text{ yd.} \times 3\frac{1}{7} = 12\frac{4}{7} \text{ yd.}$$

3. Draw circles of various diameters and find their circumferences.

In these questions we always need to know how to multiply a whole number and a fraction.

TRIANGLES

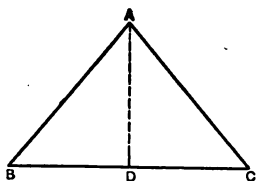


ABC is a triangle.

Its altitude is AD .

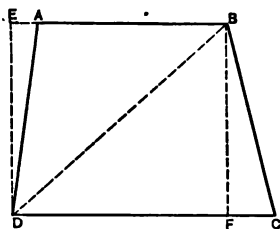
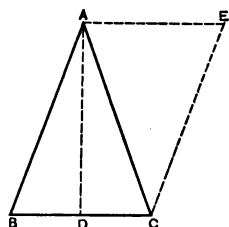
Its base is BC .

The area of a triangle = $\frac{\text{Base} \times \text{Altitude}}{2}$.



What is AC in relation to $ABCE$?

Cut out of paper the parallelogram $AECB$, and show that the triangles, AEC and ABC , are equal. What is the area of the parallelogram? What is the area of each triangle? What part of the parallelogram is either triangle?



$ABCD$ is a trapezoid. Two of its sides are *parallel*.

ED or BF is its altitude.

AB is the base of the triangle ABD .

DC is the base of the triangle DBC .

The area of $ABCD$ = area of ABD + area of DBC .

1. Draw triangles of various shapes with base 3 in. and altitude 4 in. Are all their areas equal?
2. Draw other triangles with base 6 in. and altitude 9 in. Find their areas.
3. Draw trapezoids and find their areas.

Two million nine hundred forty-five thousand one hundred sixty-seven, and five hundred eighty-three million two hundred eighty-six thousand four hundred seventeen billionths.

Read the following :

- a.* 398.7561 *d.* 278934.1 *g.* 4326915.3 *j.* .000927
b. 38.42936 *e.* 12345681.98 *h.* 986.432 *k.* 293.42865
c. 1.532943 *f.* 29.87521985 *i.* .24387 *l.* .24139876

VALUES OF DECIMALS

RECITE

$$\begin{aligned}
 a. \quad .0675 &= \frac{675}{10000} = \frac{600}{10000} + \frac{70}{10000} + \frac{5}{10000} \\
 &= \frac{6}{100} + \frac{7}{1000} + \frac{5}{10000} \\
 &= .06 + .007 + .0005 \\
 &= .0675 \\
 &= \frac{6}{\text{hundredths}} + \frac{7}{\text{thousandths}} + \frac{5}{\text{ten-thousandths}}
 \end{aligned}$$

Discuss by this method :

- b.* .349 *c.* .0028 *d.* .3486 *e.* .00387 *f.* .6036

Read each decimal as if it were a whole number, giving it the name of the order of its right-hand figure.

Read and discuss the following numbers :

- | | | | |
|-----------------|--------------------|---------------------|---------------------|
| <i>g.</i> .25 | <i>n.</i> .341 | <i>u.</i> .2374 | <i>bb.</i> 327.264 |
| <i>h.</i> .362 | <i>o.</i> .0003 | <i>v.</i> .2006 | <i>cc.</i> 80.9056 |
| <i>i.</i> .451 | <i>p.</i> 45.7056 | <i>w.</i> .0104 | <i>dd.</i> .0000008 |
| <i>j.</i> .5675 | <i>q.</i> 12.07067 | <i>x.</i> 42.05063 | <i>ee.</i> 231.146 |
| <i>k.</i> .8432 | <i>r.</i> .6914 | <i>y.</i> .00071423 | <i>ff.</i> 4.3291 |
| <i>l.</i> .003 | <i>s.</i> .008196 | <i>z.</i> 93.42 | <i>gg.</i> 71.00089 |
| <i>m.</i> .028 | <i>t.</i> .0624 | <i>aa.</i> 147.1934 | <i>hh.</i> 167.193 |

REVIEW QUESTIONS

1. Give the Multiplication Table of Twelves.
2. How can we at once write the product of any number by ten, one hundred, one thousand, etc.?
3. How many sides has a hexagon? a decagon? an octagon? Draw each on the blackboard?
4. What is the ratio of twelve apples to eighteen apples? to twenty-four? to six? to nine? to sixty?
5. Tell in the order of their sizes the following fractions:
 $\frac{1}{8}$ $\frac{1}{7}$ $\frac{1}{9}$ $\frac{1}{5}$ $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{6}$.
6. What does the denominator of a fraction tell? the numerator?
7. What kind of a fraction is $\frac{2}{7}$? $\frac{8}{7}$? $\frac{3}{5}$? $\frac{10}{4}$?
8. Change such fractions above as you think might be expressed as mixed numbers.
9. Add: 4, 3, 9, 8, 2, 6, 1, 3, 5, 8, 9, 4, 7, 4, 5.
10. What is the *quotient* in division? the *remainder*?
11. Give the aliquot parts of 100 divided in sixths.
12. Tell how we express 8% decimally.
13. A man borrowed \$10,000 at 5% a year interest. How much interest did he owe at the end of one year on each \$100? on each \$1000? on the \$10,000?
14. The tax on a house at 2% is \$20. What part of the value of the house was this? How many times the tax was the house worth? What was the house worth?
15. What are *addends*?
16. An ancient building has upon its cornerstone the letters MCDXXIV. In what year was it built?

QUESTIONS

1. A besieged town containing 15,000 inhabitants has provisions sufficient to last 5 weeks. How long will the provisions last if 7000 of the inhabitants be sent away?

2. How far can a man travel in 27 da. at the rate of 149 mi. in 4 da.?

3. If a person steps 4 yd. in 5 paces, to how many yards will 729 of his paces be equal?

4. @ 29¢ per lb., what is the cost of 174 lb. of coffee?

5. The numbers 24, 28, 32, 49, 81, are to be multiplied together and the product is to be divided by $8 \times 4 \times 7 \times 9 \times 6$. What will be the quotient?

6. Divide $16 \times 20 \times 8 \times 3$ by $30 \times 8 \times 6$.

7. How many pounds of raisins at 15¢ a pound are equal in value to 25 pounds of pepper at 48¢ a pound?

8. How much muslin at 25¢ a yard is equal in value to 100 yd. of silk at 87¢ a yard?

9. How many yards of serge at 46¢ a yard should be given for 23 bu. of wheat at 92¢ a bushel?

10. How long will it take a train to go 100 mi. at the rate of 42 mi. per hr.?

11. Add:

<i>a.</i>	<i>b.</i>	<i>c.</i>	<i>d.</i>	<i>e.</i>	<i>f.</i>	<i>g.</i>	<i>h.</i>
29	88	42	93	75	69	184	103
16	37	15	66	28	37	216	405
27	25	56	81	79	19	135	764
61	34	32	13	11	61	320	573
19	36	91	39	99	36	413	127
76	44	72	87	13	14	101	205
23	26	13	46	28	42	204	495
17	19	21	65	43	75	302	207
38	60	43	53	24	32	401	185

ADDITION OF DECIMALS

1. Add 2.0357, 120.15, .037, 19, and 70.004.

2.0357

120.15

.037

19.

70.004

211.2267

In adding decimals set the figures for units of the same order in columns as in adding numbers without decimals.

2. Add 80.007, .0091, 49.375, 1001.25, 407.7125.

80.007

.0091

49.375

1001.25

407.7125

1538.3536

Notice that the decimal points are in vertical line with each other.

3. Add by columns and by rows :

	I	II	III	IV
A	3.406	45.372	300.	4.7561
B	44.04	3650.	4.12305	.000375
C	671.75	.00003675	4120.	8.125
D	376.15	40000.	800.01725	.0125
E	43.702	30102.5	.46125	3.67003
F	4.01	.003	29.	1.2345
G	.012	32.1	4.12	.00036
H	.00023	.00967	.00476	.0391
I	39.102	.046	471.2	82.137
J	8170.007	3.295	014.	121.0258

4. Mr. Brown bought a horse for \$100, a carriage for \$172.50, a whip for \$1.375, and a carriage robe for \$3.375. What was the total amount of his purchases?

SUBTRACTION OF DECIMALS

1. Subtract 9.7658 from 117.053.

117.053

9.7658

107.2872

In subtracting decimals set the figures for units of the same order in columnus.

2. Subtract 4.12575 from 21.

21.

4.12575

16.87425

The process is this :

10 9 9 9 9

1 0 10 10 10 10 10

2 1.0 0 0 0 0

0 4.1 2 5 7 5

16.8 7 4 2 5

3. Subtract :

a. 3.594 from 13.007.

i. 5.056 from 20.91.

b. .0976 from 1.28.

j. .98976 from 1.

c. 13.265 from 100.0008.

k. 18.7346908 from 90.32064.

d. 432.125 from 678.427.

l. 412.865 from 503.125.

e. 36.75 from 40.1.

m. 4.38675 from 5.

f. 100.376 from 451.86975.

n. .000625 from 30.

g. 3.07625 from 500.8.

o. 473.5 from 1000.

h. .00061765 from .001.

p. .00384752 from 012345.

4. Find the value of :

q. 71.48 — 35.71.

t. 1.000009 — .784163.

r. .00078 — .000089.

u. 75 — .0057.

s. 87.31 — 6.871496.

v. 4.001 — .374.

5. Mr. Smith had \$875 in cash and paid as taxes on three pieces of property, \$251.654, \$401.736, and \$105.108. What amount was left?

6. Mr. Williams owned .1 A. of land and sold from it .05 A. How much land had he then?

7. \$.25 — \$.12 + \$.405 + \$.555 — \$1 = ?

DECIMALS

Express in decimal form the following fractions :

- | | |
|-----------------------------|---------------------------|
| 1. 6 tenths. | 2. 6 hundredths. |
| 3. 6 thousandths. | 4. 46 hundredths. |
| 5. 62 ten-thousandths. | 6. 674 millionths. |
| 7. 605 ten-millionths. | 8. 6901 hundredths. |
| 9. 56 thousandths. | 10. 706 thousandths. |
| 11. 376 thousandths. | 12. 625 ten-thousandths. |
| 13. 316 hundredths. | 14. 80060 thousandths. |
| 15. 306 tenths. | 16. 5046 ten-thousandths. |
| 17. 81 hundred-thousandths. | 18. 9 millionths. |

Express in decimal form the following fractions :

- | | | | |
|---------------------------|-----------------------------|-------------------------------|--------------------------|
| 19. $\frac{9}{10}$. | 20. $\frac{24}{100}$. | 21. $\frac{4}{100}$. | 22. $\frac{42}{100}$. |
| 23. $\frac{125}{1000}$. | 24. $\frac{22}{1000}$. | 25. $\frac{14}{100}$. | 26. $\frac{7}{1000}$. |
| 27. $\frac{102}{1000}$. | 28. $\frac{20}{100}$. | 29. $\frac{8}{1000}$. | 30. $\frac{30}{1000}$. |
| 31. $\frac{422}{10000}$. | 32. $\frac{296}{1000000}$. | 33. $\frac{89287}{1000000}$. | 34. $\frac{900}{1000}$. |

Some common fractions may be very easily changed to decimals. We know that a quarter of a dollar of money is twenty-five cents. Cents mean hundredths of a dollar.

$$\$ \frac{1}{4} = 25 \text{¢} = \$.25 \quad \$ \frac{1}{2} = 50 \text{¢} = \$.50 \quad \$ \frac{3}{4} = 75 \text{¢} = \$.75$$

$$\frac{1}{4} = .25, \text{ since } \frac{1}{4} \text{ of } 100 = 25, \text{ and } \frac{1}{4} \text{ of } \frac{100}{100} = \frac{25}{100}.$$

$$\frac{1}{2} = .50, \text{ since } \frac{1}{2} \text{ of } 100 = 50, \text{ and } \frac{1}{2} \text{ of } \frac{100}{100} = \frac{50}{100}.$$

$$\frac{1}{2} = .5, \text{ since } \frac{1}{2} \text{ of } 10 = 5, \text{ and } \frac{1}{2} \text{ of } \frac{10}{10} = \frac{5}{10}.$$

$$\frac{1}{5} = .2, \text{ since } \frac{1}{5} \text{ of } 10 = 2, \text{ and } \frac{1}{5} \text{ of } \frac{10}{10} = \frac{2}{10}.$$

VALUE OF ONE

$1 = \frac{10}{10}$ or $1.0 = \frac{100}{100}$ or $1.00 = \frac{1000}{1000}$ or $1.000 = \frac{10000}{10000}$
or 1.0000 , etc.

DECIMALS AND COMMON FRACTIONS

Every common fraction may be expressed in an equivalent decimal fraction, and every decimal may be expressed as a common fraction.

$\frac{1}{2} = 50$ per cent of anything, since $\frac{1}{2}$ of 100 is 50; and in percentage we think of everything as having 100 equal parts.

$$\frac{1}{4} = 25\% \quad \frac{1}{8} = 12\frac{1}{2}\% \quad \frac{1}{5} = 20\% \quad \frac{1}{10} = 10\%$$

50% may be written in decimals: $50 = .5$; $25\% = .25$; $12\frac{1}{2}\% = .125$, since $12\% = .12$, and $\frac{1}{2}$ of $.01 = \frac{1}{2}$ of $.010$, and $\frac{1}{2}$ of $.010 = .005$.

$20\% = .20 = .2$. $10\% = .10 = .1$. Since all these things we know from percentage, we can now say

$$\frac{1}{2} = .5 \quad \frac{1}{4} = .25 \quad \frac{1}{8} = .125 \quad \frac{1}{5} = .2 \quad \frac{1}{10} = .1$$

We studied all these facts when we learned the aliquot parts of 100. We are now expressing just the same facts, but in a different way.

There are a good many decimal fractions which are easy to change to common fractions.

$.1 = \frac{1}{10}$. $.2 = \frac{1}{5}$, because $\frac{2}{10} = \frac{1}{5}$. $.25 = \frac{1}{4}$. $.333+ =$ nearly $\frac{1}{3}$, since $\frac{1}{3}$ of 100 = $33\frac{1}{3}$. $.4 = \frac{2}{5}$, since $\frac{4}{10} = \frac{2}{5}$. $.5 = \frac{1}{2}$. $.666+ = \frac{2}{3}$. $.75 = \frac{3}{4}$.

1. Write the common fractions which equal .2; .333; .25; .50; .75; .1; and .375.

2. Mary had $\frac{3}{4}$ of a dollar, and spent 20 cents for a box of strawberries and 30 cents for half a quart of fine ice cream. Can you find the answer to this question by using decimals? Do so, if you think you can.

RATIOS

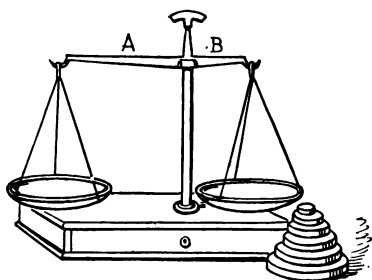


The scale arms in the first picture are supported at the center. A weight upon one of the pans is exactly counterbalanced by a weight upon the other.

One pound of the weight balances one pound of sugar.

Four ounces balance four ounces.

But the scale arms in the second picture are supported at one third of the distance from the right.



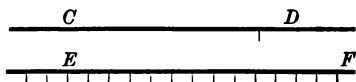
SCALES FOR WEIGHING.

$$A = 2 B.$$

The ratio of A to B is 2.

A 1-pound weight in A counterbalances 2 pounds of sugar.

The principle of the scale for weighing is ratio.



$$C = 3 D.$$

$$1 \text{ lb. in } C = 3 \text{ lb. in } D.$$

$$E = 16 F.$$

$$1 \text{ oz. in } E = 1 \text{ lb. in } F.$$

The even balance or scale as seen above shows the principle of the equation, both of whose terms are always equal.

EQUATION OF FORMS

Evidently this

$$\boxed{} \text{ equals this } \boxed{} \text{ and this } \boxed{}$$

$$A = B = C$$

If $A = B$ and $B = C$, then $A = C$.

Things equal to the same things are equal to each other.

1. If 3 apples cost 10¢ and 2 oranges cost the same as 3 apples, how much do the 2 oranges cost?
2. If John and Charles earn together \$1 a day, and Thomas earns as much as both John and Charles, how much does Thomas earn?

Evidently $\boxed{}$ equals $\boxed{}$ and $\boxed{}$

$$2A = 2B = 2C$$

3 A would equal 3 B or 3 C .

Equal multiples of equal things are equal.

3. If Mary and Sally are each 6 yr. old, when Mary is 12 years old, how old will Sally be, if she lives?
4. If one book costs \$1, how much will ten \$1 books cost?

Evidently $\boxed{}$ equals $\boxed{}$ and $\boxed{}$

$$\frac{A}{2} = \frac{B}{2} = \frac{C}{2}$$

If $\frac{A}{2} = \frac{1}{2}$ of A and $\frac{B}{2} = \frac{1}{2}$ of B , then $\frac{C}{2} = \left\{ \begin{array}{l} \frac{1}{2} \text{ of } A \\ \frac{1}{2} \text{ of } B \end{array} \right.$

Equal parts of equal things equal each other.

5. If John earns 50¢ a day for ten hours of work, how much will he earn in one fifth of a day?

RATIO AND PROPORTION

$\frac{1}{2}$ of a thing equals $\frac{2}{4}$ of the same thing. $\frac{1}{2} = \frac{2}{4}$.

$\frac{1}{4}$ of a thing equals $\frac{2}{8}$ of the same thing. $\frac{1}{4} = \frac{2}{8}$.

$\frac{1}{2}$ is a ratio. It is the ratio of 1 to 2.

$\frac{2}{4}$ is a ratio. It is the ratio of 2 to 4. • • to $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$

These ratios are equal, since $\frac{1}{2} = \frac{2}{4}$.

So we can say, The ratio of 1 to 2 equals the ratio of 2 to 4.

Expressed briefly, we say, 1 is to 2 as 2 is to 4.

We can write this: $1:2 = 2:4$.

This last form, $1:2 = 2:4$, is a **proportion**.

A proportion is an equality of ratios.

1. Explain this form: $1:4 = 2:8$.
2. Prove that $1:3 = 2:6$ and $1:5 = 2:10$.
3. Prove that $2:3 = 4:6$ and $3:4 = 6:8$.

We know that $\frac{2}{3} = \frac{4}{6}$ and that $\frac{3}{4} = \frac{6}{8}$.

4. Draw squares to show these ratios and proportions.

USEFULNESS OF PROPORTION

If we have the ratio of 2 to 4 and the number 6, and wish to know what number is to 6 as 2 is to 4, we can find it by proportion.

• • is to $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$ as what is to $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$ $2:4 = ? : 6$

We find it very helpful in arithmetic to have another sign for ?. This sign is x , the unknown quantity.

$2:4 = x:6$. Find x .

$2 = \frac{1}{2}$ of 4. $x = \frac{1}{2}$ of 6. But $\frac{1}{2}$ of 6 = 3. $x = 3$.

$2:4 = 3:6$. Proportion helps us find unknown quantities.

EQUATION OF NUMBERS

$$6 \times 5 = 3 \times 10 \quad \text{because} \quad \left. \begin{array}{l} 6 \times 5 \\ 3 \times 10 \end{array} \right\} = 30$$

$$12 \times 5 = 6 \times 10 \quad \text{By what is each quantity multiplied?}$$

$$2 \times 5 = 1 \times 10 \quad \text{By what is each quantity divided?}$$

$$2 \times 8 = 4 \times 4 \quad \text{because} \quad \left. \begin{array}{l} 2 \times 8 \\ 4 \times 4 \end{array} \right\} = 16$$

$$6 \times 8 = 12 \times 4 \quad \text{By what is each quantity multiplied?}$$

$$1 \times 8 = 2 \times 4 \quad \text{By what is each quantity divided?}$$

An equality of quantities is an **equation**.

When we have to find an unknown quantity, sometimes the equation helps us.

$$6 \times 5 = x \times 10 \quad 30 = 10x \quad \text{Divide by 10} \quad 3 = x$$

$$2 \times 8 = x \times 4 \quad 16 = 4x \quad \text{Divide by 4} \quad 4 = x$$

$$3 \times 4 = x \times 24 \quad 12 = 24x \quad \text{Divide by 24} \quad \frac{1}{2} \frac{1}{4} = x = \frac{1}{2}$$

1. John sold some apples at 2¢ each and received 24¢. How many did he sell?

$$x \times 2¢ = 24¢ \quad \text{Divide by 2¢} \quad x = 12$$

2. Mary found some buttercups on her way to school. She gave 12 to her teacher and $\frac{1}{2}$ of the rest to her school-girl friends. Then she had 3 left. How many buttercups did she find?

$$3 = \frac{x - 12}{2}, \quad \text{because } 3 = \frac{1}{2} \text{ of all she found less 12.}$$

Multiply both quantities by 2.

$$6 = x - 12 \quad \text{Add 12 to each quantity}$$

$$6 + 12 = x - 12 + 12 \quad 18 = x$$

3. John earned some money by selling papers. He gave $\frac{1}{2}$ of it to his mother, bought a cap with $\frac{1}{4}$ of it, and had 50¢ left. How much did he earn?

PROPORTION

If $2:8 = x:12$, what is x ?

$$\begin{array}{ccccccc} \frac{2}{8} = \frac{x}{12} & \frac{2}{8} = \frac{1}{4} & \frac{1}{4} = \frac{x}{12} & 1 = \frac{4x}{12} \\ \frac{4x}{12} = \frac{x}{3} & 1 = \frac{x}{3} & 3 = x & 2:8 = 3:12 \end{array}$$

The ratio of $2:8$ is $\frac{1}{4}$ and of $3:12$ is $\frac{1}{4}$.

1. How do we know that $\frac{2}{8} = \frac{1}{4}$? Show this by a drawing, if necessary.

2. Why can we say here that $\frac{1}{4} = \frac{x}{12}$? and $1 = \frac{4x}{12}$?

3. What do we think in order to make $\frac{4x}{12} = \frac{x}{3}$?

4. What truth makes us know next that $1 = \frac{x}{3}$?

5. How did we take the next step, $3 = x$?

6. Why do we write in the answer 3 for the x ?

We can always do anything to one side or member of an equation when we do the same thing to the other member.

The sign $=$ shows the two equal quantities of the equation.

a. If $3:9 = x:15$, what is x ?

b. If $2:6 = x:3$, what is x ?

c. If $2:6 = 3:x$, what is x ?

d. If $3:9 = 4:x$, what is x ?

e. John had 6 marbles and Walter had 3; William had 12 and Samuel had as many in comparison with William as Walter had in comparison with John. How many marbles had Samuel?

f. $10\% : 50\% = \$5 : \$x$. Find $\$x$.

PROPORTION

1. If a boy built a toy boat 2 inches wide and 8 inches long, and then planned to build another boat of the same shape but 12 inches long, how wide should he make it?

We can find this by proportion:

$$2:8 = x:12$$

$$\frac{2}{8} = \frac{x}{12}$$

$$\frac{2}{8} = \frac{1}{4}$$

$$1:4 = x:12$$

$$\frac{1}{4} = \frac{x}{12}$$

$$\frac{1}{4} \text{ of } 12 = 3.$$

$$2:8 = 3:12$$

The ratio of $\left\{ \begin{array}{l} 1 \text{ to } 4 \\ 3 \text{ to } 12 \end{array} \right\}$ is $\frac{1}{4}$

The boat should be 12 inches long and 3 inches wide. Draw on the blackboard these two sizes, 2×8 and 3×12 , for the boats. Do the forms seem alike in shape? Is the ratio of length to breadth in each form the same?

2. An older boy built a large boat, 30×6 inches. His brother wished to build a smaller boat, 15 inches long, to look like it. How wide did he make it?

We can find this by proportion:

$$30:6 = 15:x$$

$$\frac{30}{6} = \frac{15}{x}$$

$$\frac{30}{6} = \frac{5}{1} \text{ or } 5$$

$$5:1 = 15:x$$

$$\frac{5}{1} = \frac{15}{x}$$

$$15 = 5 \times 3$$

$$30:6 = 15:3$$

The ratio of $\left\{ \begin{array}{l} 30 \text{ to } 6 \\ 15 \text{ to } 3 \end{array} \right\}$ is 5

The boat was 3 inches wide.

3. Find the unknown x in these proportions:

a. $4:3 = x:6$.

e. $3:4 = 9:x$.

i. $5:4 = x:8$.

b. $2:3 = 4:x$.

f. $1:2 = 6:x$.

j. $2:1 = 14:x$.

c. $1:5 = 2:x$.

g. $3:5 = 12:x$.

k. $3:2 = x:10$.

d. $2:5 = x:10$.

h. $2:4 = 5:x$.

l. $2:3 = x:9$.

PERCENTAGE

When we say 10 per cent, or write 10%, we mean 10 : 100. 15% means 15 : 100, or $\frac{15}{100}$. What do 25%, $12\frac{1}{2}\%$, 30%, 100%, 75%, mean?

Percentage is a ratio of one thing to some other thing, supposed to be divided into 100 parts.

1. A boy had \$3 and gave 15% to his brother. How much did he give him?

We can find the answer by proportion and equation.

$$\begin{array}{ll} \text{dollars} & \text{per cent} \\ x : 3 = 15 : 100 \end{array}$$

$$\frac{x}{3} = \frac{15}{100}$$

Multiply by 3

$$x = \frac{45}{100}$$

$$\frac{45}{100} \text{ dollars} = \$.45 = 45 \text{¢}$$

2. Another boy had 200 pigeons. He gave 20% to a friend and sold 30% to a neighbor. How many had he left?

$$20\% + 30\% = 50\%$$

$$\begin{array}{ll} \text{pigeons} & \text{per cent} \\ x : 200 = 50 : 100 \end{array}$$

$$\frac{50}{100} = \frac{x}{200}$$

Multiply by 100

$$50 = \frac{x}{2}$$

Multiply by 2

$$100 = x$$

3. Find the unknown quantity :

a. 20% of $x = 50$.

f. 10% of $x = \$2$.

b. 15% of $x = 30$.

g. 5% of $x = \$200$.

c. $12\frac{1}{2}\%$ of $x = 4$.

h. 50% of $x = \$1200$.

d. $66\frac{2}{3}\%$ of $x = 20$.

i. 75% of $x = \$750$.

e. 25% of $x = 5$.

j. $33\frac{1}{3}\%$ of $x = \$1000$.

4. A man borrowed some money at 6% interest for a year. At the end of the year he paid \$12 as interest. How much money did he borrow?

REVIEW

1. If the circumference of a wheel is 16 feet, how many feet will the wheel travel in 375 revolutions?

2. If a train moves at the rate of 43 miles an hour, how many miles will it travel in 113 hours?

3. How many cubic inches are there in 23 cubic feet? in 40 cubic feet? in 50 cubic feet?

4. A city containing 24,500 inhabitants has provisions for 17 weeks. How many people could live for one week on those provisions?

5. If a man spends \$13 a week, how many dollars will he spend in 52 weeks? in 104 weeks?

6. If a man can walk 4 miles an hour, how many miles can he walk in 13 hours?

7. How many cubic feet are there in 391 cubic yards? in 400 cubic yards?

8. If one man can lift 521 pounds' weight, how many pounds can 357 equally strong men lift?

9. If a man walks 37 miles in a day, how many miles does he walk in 313 days? in 365 days?

10. If 39 men can do a certain amount of work in 23 days, how many days will one man take to do the work?

11. If an acre of wheat yields 29 bushels, how many bushels will 149 acres yield?

12. What will 217 horses cost at \$106 each?

13. If a city contains 97 streets, and if there are, on an average, 304 houses on each street and 7 persons in each house, what is the population of the city?

14. Since sound travels about 1100 feet per second, how far does it travel in 5 seconds?

VALUE OF MONEY

1. Make a list of things and services money will buy. Let each boy and girl in the class make suggestions. The articles may be classified as Necessities, Comforts, Luxuries; the services as Business, Professional, Domestic.

2. Make lists of the things one dollar will buy; one quarter; one dime; a ten-dollar bill.

3. Develop the ratio of money to articles and services.

\$1 buys a nice book, a boy's hat, six pounds of beef.

\$2 pays for the doctor's visit, for an 80-mile railroad ride.

\$3 buys a pair of men's shoes, or pays the dentist for filling a tooth.

\$4 buys a parlor chair, or rents a piano for a month.

\$5 buys a girl's dress, or a small boy's suit of clothes.

\$20 pays a month's rent for a six-room house "with improvements."

\$100 buys a horse, or pays a 2% tax on a \$5000 house.

4. Show \$1, \$2, \$5, \$10 bills to the pupils; also the \$10 gold eagle; and ask their ratios to small coins.

5. Discuss penny-and-nickel-a-day saving and its results; also discuss penny-a-day spending for trifles.

6. Explain savings bank interest: \$3, \$3½, or \$4 a year for each \$100.

7. Discuss good books as good investments even of money. Explain even the money-earnings of such investments.

8. \$1000 a year is a fair income for a man who has had 15 years' education. \$500 a year is a fair income for a man with less than 5 years' education. \$500 is 5% interest on what amount of money? Why is a good education usually worth at least as much as \$25,000?

Call these subjects "the purchasing power of money."

REVIEW

1. Two men travel in opposite directions ; one goes 31 miles a day, and the other 42. How far apart will they be in 12 days ?

2. How long can 6 men live on an amount of food that will last one man in the Arctics 762 days ? Would an exploration party find it useful to know these facts ?

3. Five hundred four dollars are to be divided equally among seven men. How many dollars should be given to each man ?

4. A tank contains 16,632 cubic inches of water. How many gallons of water are in the tank ?

5. With horses at \$100 each, cows at \$50 each, sheep at \$12.50 each, how many of these different animals will \$1000 buy, if one buys all of the same kind ?

6. If $3 : 15 = x : 20$, what is x ?

7. Wood was \$4 a cord. A dealer paid for \$1000 worth. How many cords did he buy ?

8. In a battle in Cuba, 67 soldiers were killed out of 823 engaged. How many men were left alive ?

9. A steamer travels 10,082 miles in one year, and 8241 in a second year. How many more miles did the steamer travel in the first year than in the second ?

10. A boy filled ninety-nine boxes with oranges, packing in each box three hundred ninety-nine oranges. How many oranges did he pack in all the boxes ?

11. A room was 12 ft. high, 24 ft. wide, and 32 ft. long. How many cubic feet did it contain ?

12. If each boy breathed 30 cu. ft. of air a minute, how long would the air be fresh with one boy in the room ? with 50 boys ?

13. If $2 : 9 = 4 : x$, what is x ?

MONEY AND TAXES

Mills are not coined. But they have very important duties to perform. All the people must help support the **government**, which rules **our country** for us. We have to pay for the government buildings, and for the services of those engaged in government. We raise the money for all government purposes by **taxes** upon the wealth of the citizens. The wealth that people own themselves is called **property**. We usually count taxes as so many dollars, cents, and mills upon every hundred dollars' worth of property.

One State allows towns and cities to tax every \$100 worth of property three mills to support a public library. A man with a house worth \$3100 would have to pay 31 times 3 mills or 93 mills a year for the support of the public library, if his town had such a public library. 93 mills = 9.3¢. Mills help us to arrange for very small amounts of tax from a great many different people; yet all these amounts together make a great deal of money.

A town with a 3-mill tax for a public library would get \$3000 a year from \$10,000,000 of property.

COINAGE

All the metal money of the United States is coined in the Government Mints. Of these Mints the largest is in the city of Philadelphia. Do you know where Philadelphia is? Do you know anything about Philadelphia? What do you know about our Mints?

1. How many cents or hundredths of a dollar are there in \$1.025? How many mills or thousandths of a dollar?
2. What is the difference between \$2½ and 75¢?

REVIEW

1. If a man works nine hours a day, how many hours does he work in 301 days?

2. A boy traveled in 7 days in a train 2870 miles. How far did he travel each day?

3. If 8 men can do a piece of work in 10 days, in how many days and parts of days can 24 men do it working at the same rate?

4. A farmer stored in his barns 100 tons of hay for the winter. He kept 40 cows and horses. At the end of the winter 40 tons were left. How many tons each did the cows and horses eat on the average?

5. A man had \$500 to lend. One person offered him 4% on it, and another offered 5%. How much money, more or less, would he make in one year by lending at the 5% rate?

6. What is a proportion? an equation?

7. Add .02, .25, .13.

8. Subtract 1.29 from 2.453.

9. What decimal equals the common fraction $\frac{1}{8}$?

10. Read 1001.2324.

11. Give the tables for counting, time, avoirdupois weight, and dry and liquid measures.

12. What is the circumference of a circle 10 ft. in diameter?

13. What is the area of a right-angled triangle 5 ft. by 24 ft.?

14. How much is 5×6 divided by 2×3 ?

QUESTIONS

1. What will 8 yards of sheeting cost at $12\frac{1}{2}\text{¢}$ a yard?
2. What will 9 pounds of prunes cost at 9¢ a pound?
3. What is the cost of 7 lb. of butter at 22¢ per lb.?
4. What is the cost of 12 pounds of tea at 60¢ a pound?
5. What is the cost of 12 lb. of coffee at 30¢ per lb.?
6. What is the cost of 11 yards of cloth at $\$4$ a yard?
7. What is the cost of 9 books at 40¢ each?
8. At $\$12$ a cord, what will 12 cords of wood cost?
10 cords? 9 cords? 7 cords? 6 cords? 5 cords?
9. How many feet are there in 11 yards? 10 yards?
29 yards? 48 yards? 97 yards?
10. How many quarts are there in 10 gallons? 9 gal-
lons? 8 gallons? 50 gallons?
11. If you buy 5 marbles for 1¢ , how many marbles
can you buy for 11¢ ? 8¢ ? $\$1.00$?
12. At 8¢ a quart, what will 12 quarts of milk cost?
11 quarts? 6 quarts? 24 quarts? 98 quarts? 17 quarts?
13. What will be the cost of 12 yards of braid at 10¢ a
yard? 10 yards? 60 yards? 8 yards? 25 yards?
14. What is the cost of 8 yards of silk at $\$5$ a yard?
15. If 7 yards of cloth make a suit, how many yards
will it take to make 8 suits?
16. How much do 9 pounds of ginger cost at 30¢ a
pound?
17. At $\$12$ apiece, what will 10 sofas cost?
18. What is the cost of 10 bbl. of vinegar at $\$9$ per bbl.?
19. What will 11 pair of shoes cost at $\$5$ a pair?
20. If 8 men can build a wall in 9 days, how long will
it take 1 man to build it?

MULTIPLICATIONS

	A	B	C	D
E	$2 \times 2 = ?$	$3 \times 2 = ?$	$2 \times 4 = ?$	$3 \times 6 = ?$
F	$4 \times 3 = ?$	$5 \times 3 = ?$	$4 \times 5 = ?$	$5 \times 7 = ?$
G	$6 \times 4 = ?$	$7 \times 4 = ?$	$6 \times 6 = ?$	$7 \times 9 = ?$
H	$8 \times 5 = ?$	$9 \times 5 = ?$	$8 \times 7 = ?$	$9 \times 8 = ?$
I	$10 \times 6 = ?$	$11 \times 6 = ?$	$10 \times 8 = ?$	$11 \times 7 = ?$
J	$12 \times 7 = ?$	$13 \times 7 = ?$	$12 \times 9 = ?$	$13 \times 6 = ?$
K	$14 \times 7 = ?$	$15 \times 7 = ?$	$14 \times 9 = ?$	$15 \times 5 = ?$
L	$16 \times 6 = ?$	$17 \times 6 = ?$	$17 \times 8 = ?$	$17 \times 4 = ?$
M	$18 \times 5 = ?$	$19 \times 5 = ?$	$18 \times 7 = ?$	$19 \times 3 = ?$
N	$20 \times 4 = ?$	$21 \times 4 = ?$	$20 \times 6 = ?$	$21 \times 2 = ?$
O	$22 \times 3 = ?$	$23 \times 3 = ?$	$22 \times 5 = ?$	$23 \times 3 = ?$
P	$24 \times 2 = ?$	$25 \times 2 = ?$	$24 \times 4 = ?$	$25 \times 4 = ?$
Q	$12\frac{1}{2} \times 8 = ?$	$16\frac{2}{3} \times 6 = ?$	$33\frac{1}{3} \times 3 = ?$	$37\frac{1}{2} \times 2 = ?$
R	$48 \times 2 = ?$	$20 \times 5 = ?$	$8\frac{1}{3} \times 12 = ?$	$66\frac{2}{3} \times 3 = ?$
S	$12\frac{1}{2} \times 2 = ?$	$16\frac{2}{3} \times 3 = ?$	$12 \times 8 = ?$	$100 \times \frac{1}{2} = ?$
T	$3 \times 3 = ?$	$4 \times 4 = ?$	$5 \times 5 = ?$	$7 \times 7 = ?$
U	$8 \times 8 = ?$	$9 \times 9 = ?$	$12 \times 12 = ?$	$11 \times 11 = ?$
V	$15 \times 15 = ?$	$18 \times 18 = ?$	$20 \times 20 = ?$	$24 \times 24 = ?$
W	$25 \times 25 = ?$	$50 \times 50 = ?$	$100 \times 100 = ?$	$250 \times 4 = ?$

EASY METHODS

1. $98¢ \times 9 = ?$ $100¢ - 98¢ = 2¢$ $100¢ \times 9 = 900¢$
 $2¢ \times 9 = 18¢$ $900¢ - 18¢ = 882¢ = \$8.82.$

2. $25¢ \times 35 = ?$ $25¢ \times 4 = 100$ $100¢ \times 35 = 3500¢$
 $3500¢ \div 4 = 875¢ = \$8.75.$

3. $49¢ \times 10 = ?$ $49¢ = \$.49$ $\$.49 \times 10 = \$4.9 =$
 $\$4.90.$

4. $4\frac{1}{2}¢ \times 14 = ?$ $4\frac{1}{2}¢ = 9¢ \div 2$ $9¢ \times 14 = 126¢$
 $126¢ \div 2 = 63¢$ or $4\frac{1}{2}¢ \times 2 = 9¢$ $14 \div 2 = 7$
 $4\frac{1}{2}¢ \times 14 = 9¢ \times 7 = 63¢.$

DECIMAL FRACTIONS

The **powers** of a number are its products when multiplied one or more times by itself. The *first power* is the number itself.

$2 \times 2 = 4$. 4 is the second power of two.

The **powers** of 10 are 10, 100, 1000, 10,000, etc. These are called the *first, second, third, fourth . . . powers* of ten.

$10 \times 10 \times 10 = 1000$. 1000 is the third power of 10.

A fraction which has for its denominator one of the powers of 10 is called a **decimal fraction**, or a **decimal**. All other fractions are called *common fractions*.

To simplify the writing of the denominators of decimal fractions, a method of notation is used by which we express the value of the denominator in every case by the decimal form of writing the numerator.

.3 stands for $\frac{3}{10}$, and is read three tenths.

.25 stands for $\frac{25}{100}$, and is read twenty-five hundredths.

.347 stands for $\frac{347}{1000}$, and is read three hundred forty-seven thousandths.

In each case the figures that follow the *decimal point* form the numerator of the fraction.

The number of figures that follow the *point* corresponds to the number denoting the particular power of 10 that forms the denominator of the fraction.

As the first power of 10 is 1 followed by zero, and the second power of 10 is 1 followed by two zeros, and the third power of 10 is 1 followed by three zeros, etc., in every case we can write the denominator by affixing to 1 a number of zeros equal to the number of figures that follow the decimal point.

.426789 stands for $\frac{426789}{1000000}$.

Six zeros are affixed to the 1, because the number of figures that follow the decimal point is in this case six. This number is read four hundred twenty-six thousand seven hundred eighty-nine millionths.

$$\frac{275}{1000} = .275$$

$$\frac{36}{1000} = .036$$

$$\frac{743}{1000000} = .00743$$

.00025 stands for $\frac{25}{1000000}$.025707 stands for $\frac{25707}{1000000}$

The ciphers that come between the point and the figures 25 are not written in the numerators of the common fractions, for 00025 equals 25, and 025707 equals 25707.

The decimal point changes the value of figures.

Any common fraction, having as a denominator a power of 10, can be written as a decimal, by writing the numerator only, and so placing the decimal point that it shall have as many figures on the right hand as there are ciphers in the denominator of the given fraction. If the numerator does not contain as many figures as there are ciphers in the denominator, the lack must be supplied by placing as many ciphers between the point and the given numerator as are needed.

1. Write as decimals :

a. $\frac{3}{10}$.	d. $\frac{9}{10000}$.	g. $\frac{27}{100}$.	j. $\frac{3287}{100}$.	m. $\frac{319}{1000}$.
b. $\frac{5}{100}$.	e. $\frac{38}{1000}$.	h. $\frac{47}{1000000}$.	k. $\frac{4631}{1000}$.	n. $\frac{4374}{10000}$.
c. $\frac{7}{1000}$.	f. $\frac{502}{1000}$.	i. $\frac{86532}{100}$.	l. $\frac{976}{100000}$.	o. $\frac{397}{10}$.

2. Change to common fractions :

p. .079	r. .007	t. .00003	v. .15698	x. .8	z. .193
q. .0321	s. .5146	u. .0078	w. .000301	y. .0099	g. .00307

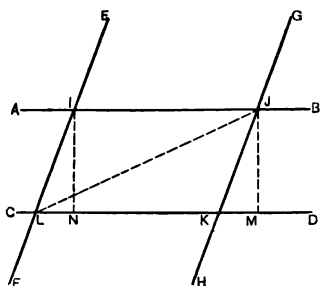
3. Decimal means tenth. Why is it a suitable word for this kind of fractions?

4. How many times do we multiply ten by itself to produce 10,000? 1,000,000? 1,000,000,000?

FIGURES

Lines that can never meet however far they may be extended are **parallel**.

When two pairs of parallel lines cross each other, the figure made by them is called a **parallelogram**.



AB and *CD* are *parallel*.

EF and *GH* are *parallel*.

The pairs of parallels cross.

What is the figure *IJKL*?

The line connecting the opposite angles of a parallelogram is a **diagonal**. What is *IJ*?

A perpendicular line between the parallels of a parallelogram gives its **altitude**. What is the *altitude* of *IJKL*?

The *rectangle IJMN* is equal in *area* to the *parallelogram IJKL*.

Draw these figures on paper. Cut out the triangle *JMK*. Does it equal the triangle *ILN*? If the figure *IJKN* is a part of both *IJMN* and *IJKL*, what does the equality of the triangles show?

The *area* of any *rectangle* equals the product of the numbers that measure its *length* and *breadth*.

The *area* of any *parallelogram* equals the product of the numbers that measure its *base* and *altitude*.

The *base* of a parallelogram is either of the parallel sides between which the *altitude* is taken.

LK or *IJ* is the *base* of *IJKL*.

Draw these parallelograms, and find their altitudes, bases, and areas:

- | | | |
|-------------------------|--------------------------|--------------------------|
| 1. 6 in. $\times$ 2 in. | 2. 5 in. $\times$ 3 in. | 3. 8 in. $\times$ 4 in. |
| 4. 9 in. $\times$ 4 in. | 5. 10 in. $\times$ 2 in. | 6. 12 in. $\times$ 5 in. |
| 7. 8 in. $\times$ 6 in. | 8. 12 in. $\times$ 4 in. | 9. 15 in. $\times$ 8 in. |

Do not find the altitudes by measurements with fractions smaller than $\frac{1}{8}$ in. Draw some rectangles among these parallelograms.

THE DECIMAL NOTATION

TEACH

A number containing decimal units is called a decimal number or a decimal. The part at the right of the decimal point is called the **decimal** part; the part at the left is termed a **whole number** or **integer**.

In the number 6666, the four figures, beginning at the left and counting toward the right, have respectively these values: 6000, 600, 60, 6; the first 6 at the left having 10 times the value of the second; the second 6, 10 times the value of the third 6; etc. The values of the figures, counting toward the right, decrease in a tenfold ratio. But, beginning at units' place and counting toward the left, each figure has only one tenth of the value of the figure to the left of it. Any figure moved one place to the left is multiplied by 10, and any figure moved one place to the right is divided by 10.

When this notation is continued to the right of the decimal point, beyond units' place, as in 6666.6666 the value of the first figure to the right of units' is $\frac{1}{10}$ of 6, or $\frac{6}{10}$; the value of the next figure is $\frac{1}{10}$ of $\frac{6}{10}$, or $\frac{6}{100}$; the value of the next figure is $\frac{6}{1000}$; of the next figure $\frac{6}{10000}$.

The figures on the right of the decimal point may be written as common fractions, $\frac{6}{10}$, $\frac{6}{100}$, $\frac{6}{1000}$, $\frac{6}{10000}$, each of these having as a denominator some power of 10.

$$\frac{6}{10} = .6 \quad \frac{6}{100} = .06 \quad \frac{6}{1000} = .006 \quad \frac{6}{10000} = .0006$$

ZEROS IN DECIMALS

$$.3 = \frac{3}{10}$$

$$.03 = \frac{3}{100}$$

$$.003 = \frac{3}{1000}$$

$$.8 = \frac{8}{10}$$

$$.0008 = \frac{8}{10000}$$

$$8.070 = \frac{807}{100}$$

But $.3 = .30 = .300 = .3000 = .30000$,

since $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000} = \frac{3000}{10000} = \frac{30000}{100000}$.

and $003 = 03 = 3 = 3.0 = 3.00$,

since in each case we have three units and three only.

In a decimal fraction a zero between the *decimal point* and any digit of value at the right of the *point* shows the order of the digit and absence of value in the order filled by the zero.

Explain the zeros in :

I. .03507 II. .00581 III. .5700 IV. .901306

MULTIPLYING AND DIVIDING BY TEN

$$.3 \times 10 = 3 \text{ since } \frac{3}{10} \times 10 = 3$$

a. $.5 \times 10 = ?$ b. $.8 \times 10 = ?$ c. $.7 \times 10 = ?$

$$.65 \times 10 = 6.5 \text{ since } \frac{65}{100} \times 10 = \frac{65}{10} = 6\frac{5}{10}$$

d. $.89 \times 10 = ?$ e. $.54 \times 10 = ?$ f. $.93 \times 10 = ?$

g. $? \times 10 = 5.4$ h. $? \times 10 = 8.7$ i. $? \times 10 = 5.8$

$$.5 \div 10 = .05 \text{ since } \frac{1}{10} \text{ of } \frac{5}{10} = \frac{5}{100}$$

j. $.9 \div 10 = ?$ k. $.6 \div 10 = ?$ l. $3 \div 10 = ?$

In decimals notice always the **DECIMAL POINT**.

READING DECIMALS

By the rule of decimal notation a digit one place to the right of the point has ten times the value of the same digit two places to the right.

.5 or 5 tenths = 50 hundredths or 500 thousandths, and .596 may be read in any of the following ways :

$$.596 = \begin{cases} 5 \text{ tenths, 9 hundredths, and 6 thousandths;} \\ \text{or 59 hundredths and 6 thousandths;} \\ \text{or 5 tenths and 96 thousandths;} \\ \text{or 596 thousandths.} \end{cases}$$

MONEY AND DECIMALS

1. Express decimally :

- | | |
|---|---|
| <p><i>a.</i> \$17 and 5 mills.</p> <p><i>b.</i> \$215 and 8 cents.</p> <p><i>c.</i> \$275 and 5 mills.</p> <p><i>d.</i> \$9 and 8 mills.</p> <p><i>e.</i> \$15 and 6¢ 9 m.</p> <p><i>f.</i> \$27 and 18¢ 2 m.</p> <p><i>g.</i> \$39 and 39¢ 7 m.</p> <p><i>h.</i> \$37 and 26¢ 7 m.</p> | <p><i>i.</i> 4 dollars and 6 mills.</p> <p><i>j.</i> 109 dollars and 1 mill.</p> <p><i>k.</i> 65 cents and 2 mills.</p> <p><i>l.</i> \$12 and 3 mills.</p> <p><i>m.</i> \$147 and 4 cents.</p> <p><i>n.</i> \$148 and 4 mills.</p> <p><i>o.</i> \$4 and 6 mills.</p> <p><i>p.</i> \$9 and 6¢ 9 m.</p> |
|---|---|

2. How many mills are there in 67 cents? in \$54?

3. How many cents are there in \$125? in \$1000?

4. In \$400 are how many cents? how many mills?

5. In \$375 are how many cents? how many mills?

6. How many mills are there in \$4? in \$6000? in \$10?

7. In \$71 45 cents 3 mills are how many mills in all?

8. Change :

- a.* 1607 mills to dollars, cents, and mills.
- b.* 170464 cents to dollars and cents.
- c.* 94780900 cents to dollars and cents.
- d.* 74164210 mills to dollars, cents, and mills.

9. Read the following :

.2	.306	.3551	2.02004
.02	.62854	.0986	121.300387
.002	.00016	.82008	2.3158218

DECIMALS

Write in figures :

1. Forty-one, and three tenths.
2. Sixteen, and three millionths.
3. Sixty-five, and fifteen thousandths.
4. Eighty, and three millionths.
5. Two, and three hundred millionths.
6. Four hundred, and ninety-two thousandths.
7. Three thousand, and twenty-one ten-thousandths.
8. Forty-seven, and twenty-one hundred-thousandths.
9. Fifteen hundred, and three millionths.
10. Thirty-nine, and six hundred forty thousandths.
11. Three thousand, and eight hundred forty millionths.
12. Six hundred, and fifteen thousandths.
13. Fifty-eight ten-thousandths.
14. One thousand, and nine hundred-thousandths.
15. Seven ten-thousandths.
16. 300 units, and three hundred-thousandths.
17. Seven hundred, and four millionths.
18. Nine thousand, and fifty-six millionths.
19. Sixty units, and 502784 millionths.
20. Six hundred thousand, and four millionths.
21. Twenty-seven, and four tenths.
22. Thirty-six, and fifteen thousandths.
23. Ninety-nine, and twenty-seven ten-thousandths.
24. Three hundred twenty thousandths.
25. Two hundred, and three hundred twenty millionths.
26. Three thousand six hundred ten ten-thousandths.

27. Read the following :

<i>a.</i> .32	<i>f.</i> 23.07	<i>k.</i> 41.603	<i>p.</i> 22.3628
<i>b.</i> .246	<i>g.</i> 12.683	<i>l.</i> 1.2864	<i>q.</i> 105.26
<i>c.</i> .3624	<i>h.</i> 20.064	<i>m.</i> 1000.115	<i>r.</i> 1000.3
<i>d.</i> 42.068	<i>i.</i> 35.0072	<i>n.</i> 871.29875	<i>s.</i> .2468953
<i>e.</i> 17.401	<i>j.</i> 67.4008	<i>o.</i> 2.3003	<i>t.</i> 10.000001

28. Write without denominators :

WRITE

<i>a.</i> 462 thousandths.	<i>g.</i> 2391 ten-thousandths.
<i>b.</i> 25 hundred-thousandths.	<i>h.</i> 65 hundred-millionths.
<i>c.</i> 1637246 ten-millionths.	<i>i.</i> 7 millionths.
<i>d.</i> 71 thousandths.	<i>j.</i> 19 ten-thousandths.
<i>e.</i> 261 hundred-thousandths.	<i>k.</i> 65 hundredths.
<i>f.</i> 751 trillionths.	<i>l.</i> 121 millionths.

29. Write in figures :

- Forty-nine hundred ten-thousandths.
- Fifty-nine, and sixty-seven ten-thousandths.
- Four hundred sixty-nine ten-thousandths.
- Seventy-nine, and four hundred fifteen millionths.
- One hundred five, and ninety-five ten-millionths.

OUR DECIMAL MONEY

STUDY

In United States money the dollar is the unit, dimes are tenths of a dollar, cents hundredths, and mills thousandths ; hence, any sum may be expressed in dollars and decimals of a dollar.

\$5 and 4 cents are \$5 and $\frac{4}{100}$; written also \$5.04.

\$20 and 96 cents and 8 mills are \$20, 96 hundredths and 8 thousandths of a dollar ; written \$20.968.

The mill is not coined. It is used only in computation, or "figuring." 5 mills or more count as a cent.

REDUCTION OF DECIMALS

Annexing ciphers to a decimal does not alter its value.

1. Reduce 9 to tenths. 9.0 Read, *ninety tenths*.

2. Express 2.1 as thousandths. 2.100. Read, *two thousand one hundred thousandths*.

Two thousand *and* one hundred thousandths would mean 2000.100.

3. Write thirty-four hundredths as ten-thousandths. .3400. Read, *three thousand four hundred ten-thousandths*.

4. Reduce to ten-thousandths :

a. .87 c. .657 e. .06 g. .325 i. .334

b. .015 d. .17 f. .77 h. .087 j. .009

5. Reduce to millionths :

k. .7208 m. .9361 o. .0035

l. .7051 n. 1.327 p. 15.00789

6. Reduce to hundred-thousandths :

a. .246 f. 23.07 k. 67.4008 p. 100.024

b. .3624 g. 3.072 l. 42.068 q. 1.0024

c. 12.683 h. .124 m. 17.401 r. 6.0009

d. 20.064 i. .35 n. 47.0025 s. 7011.2

e. 2.009 j. 35.0072 o. 53.734 t. 80.0056

RECITE

7. What part is .09 of .9? .007 of .7? .08 of 8?

8. What is the ratio of .6 to 6? of 9 to 9?

9. What is the ratio of 1 to .01? of .3 to .03?

10. What is the purpose of the zero in .305? 4.50?

11. What per cent is \$.9 of \$9? \$.4 of \$.8?

12. What fraction corresponds with per cent?

13. What is 10 % of 1 expressed decimally? of .01?

14. Read the following :

a. .1 qt. b. .3 cd. c. .08 mi. d. .5 pk. e. \$.01.

DECIMALS IN REVIEW

1. Find the sum of:

- a. 43.006, .73, .0006, 4.
- b. 7.89, 46.306, .000674, .001.
- c. 17.00013, 360, .007, 4.09.
- d. 19.0406, 173.00009, 3.5, 1000.
- e. 92.183, .706302, 400, 1.7496, .0406739.
- f. 100.01, 1.0001, 1000.0034, 10000, 88898.9865.
- g. .0698, 30.74, 2076.940635, .00473296, 1.09, 400.06.
- h. 13.97643, .00917, 4.96, 31.7049631, 4789, .3406.
- i. 1400, 3.003, .79867, 512.6345, .000987, 99.98.
- j. 55.55, .005555, .5555, .555, 5.00005.

2. Answer the following:

- k. $478.63 + 89.005 + 5437.25 + .000125 + 541.2375 = ?$
- l. $6.75 + 89.5 + .00009 + .502 + 7.548 + 2178 = ?$
- m. $51.416 + 915.13 + .04751 + .005819 + .000045.$
- n. $74.874 + 81.33486 + .0974 + 1.0718 + 88.71034 = ?$
- o. $61.371568 + .0091 + .06371 + .0008714 + 43.814 = ?$
- p. $75.67 + 125 + .00725 + 67 + 19.25 + 37 + 2145.0007 = ?$

3. Find the value of:

- q. $91.75 + 543.175 - 89.005 - 17.007 + 161.1003 - 317.$
- r. $17.48 - 1.586 + .374 + .00076 - 41.9 + 157.01.$
- s. $.0078 - .172 - .78154 + 37.011 - 11.9901 + 169.999.$
- t. $51.16 - .0073 + 89.4 - 3.90909 + 41.051 - 2.09091.$
- u. Multiply .15 by 100 and divide the product by .025.
- v. From 10 subtract a thousandth part of 1.
- w. Subtract a thousandth part of 10 from its hundredth part.
- x. If the sum of 1.015 and .001015 be added to their difference, how much will the total fall short of 1000?
- y. Find the perimeter of a square whose side is 3.25 in.

MULTIPLICATION OF DECIMALS

Multiply 87.7 by 2.03.

$$\begin{array}{r}
 87.7 \\
 2.03 \\
 \hline
 2.631 \\
 175.4 \\
 \hline
 178.031
 \end{array}$$

This result may be proved to be correct by reducing both factors to common fractions

$$87.7 \times 2.03 = \frac{877}{10} \times \frac{203}{100} = \frac{178031}{1000} = 178.031.$$

The number of ciphers in the denominator of the product is equal to the sum of the numbers of ciphers in the denominator of each factor, and the number of ciphers is equal to the number of decimal places; therefore, the number of decimal places in the product must be equal to the sum of the number of decimal places in both factors.

Multiply .0071 by .012.

$$\begin{array}{r}
 .0071 \\
 .012 \\
 \hline
 .0000852
 \end{array}$$

$$\text{Or, } .0071 \times .012 = \frac{71}{10000} \times \frac{12}{1000} = \frac{852}{10000000} = .0000852.$$

Here, since there are seven ciphers in the denominator, seven decimal places must be marked off in the decimal fraction. Since there are but three figures indicating units of value, four ciphers must be prefixed to make up the required number of places.

In multiplying decimals we point off in the product as many decimal places as the sum of the decimal places in both multiplicand and multiplier.

Find the products:

- | 1. | 2. | 3. |
|---------------------------|-----------------------------|-------------------------------|
| a. $.012 \times .4$. | a. $150 \times .05$. | a. 25.6×25 . |
| b. $101 \times .001$. | b. $.925 \times 2.5$. | b. $140.03 \times .007$. |
| c. $.1 \times 20$. | c. $.240 \times .003$. | c. $.0029 \times 2.51$. |
| d. $.032 \times 80$. | d. $.024 \times 300$. | d. 1.011×10.11 . |
| e. $50 \times .25$. | e. $1.1 \times .1$. | e. $.59 \times 83$. |
| f. $.018 \times .03$. | f. $100 \times .001$. | f. 8.7651×30 . |
| g. $.625 \times .05$. | g. $400 \times .04$. | g. $.0025 \times 1240$. |
| h. $.001 \times 1.01$. | h. $732 \times .625$. | h. 100.001×100.1 . |
| i. 30.5×7.5 . | i. $.769 \times 2900$. | i. 3.10459×3.14159 . |
| j. $47.001 \times .304$. | j. $1.153825 \times .404$. | j. $34.736 \times .00073$. |

4. In 1 rod there are 16.5 ft. How many feet are there in 41.3 rd.? In 48.75 rd.?

5. In 1 barrel there are 31.5 gal. How many gallons in 65.25 bbl.? In 100 bbl.?

6. In 1 square rod there are 30.25 square yards. How many square yards are there in 26.05 square rods?

7. In 1 square rod are 272.25 sq. ft. How many square feet are in 160 sq. rd.? in 500 sq. rd.?

Find the products:

- | 8. | 9. |
|---|---|
| a. $17.406 \times 3.7 \times .01$. | a. $5.4 \times 7.3 \times .5$. |
| b. $10.3 \times .011 \times 15.99$. | b. $4.93 \times 13.637 \times 12$. |
| c. $17.39 \times .00067 \times 13.98$. | c. $147.6984 \times .00697$. |
| d. $.2 \times .2 \times .2$. | d. $3.7 \times 4.73 \times .001$. |
| e. $30.92 \times 2.07 \times 51$. | e. $1000 \times .2 \times .05$. |
| f. $1.01 \times 10.1 \times 101 \times 101$. | f. $.05 \times .005 \times .5 \times 500$. |

10. What is the area of a parallelogram, whose base is 176.19 ft. and altitude 142.38 ft.?

DIVISION OF DECIMALS

1. Divide .285 by 1.5.

$$\begin{array}{r}
 1.5 \overline{) \overset{\vee}{.285}} (.19 \\
 \underline{15} \\
 135 \\
 \underline{135} \\
 0
 \end{array}$$

Since the dividend is the product of the divisor and quotient, the number of decimal places in the quotient must equal the excess of the number of decimal places in the dividend

over the number of places in the divisor.

The carat $\vee$ shows the excess in the problem.

The correctness of the work may be tested by reducing the decimals to common fractions and dividing.

$$\begin{array}{l}
 .285 = \frac{285}{1000} \\
 1.5 = \frac{15}{10}
 \end{array}
 \qquad
 \frac{285}{1000} \div \frac{15}{10} = \frac{285}{1000} \times \frac{10}{15} = \frac{19}{100} = .19$$

2. Divide 114.375 by 1.525.

$$\begin{array}{r}
 1.525 \overline{) 114.375} \overset{\vee}{(.75} \\
 \underline{106 } \\
 7 \\
 \underline{7 } \\
 0
 \end{array}$$

Why are there no decimal places in the quotient?

$$\text{Or, } 114.375 \div 1.525 = \frac{114375}{1000} \times \frac{1000}{1525} = \frac{114375}{1525} = 75$$

3. Divide .175 by 40.

$$\begin{array}{r}
 40 \overline{) .175000} \overset{\vee}{(.4375} \\
 \underline{160} \\
 150 \\
 \underline{120} \\
 300 \\
 \underline{280} \\
 200 \\
 \underline{200} \\
 0
 \end{array}$$

In decimals a smaller number may be divided by a larger.

Why is it necessary to add the ciphers before the figure 4 in the quotient?

4. Divide:

- | | |
|-----------------------|-------------------------|
| a. 228.75 by 7.5. | o. 3.1416 by .7854. |
| b. 1.96042 by 140.03. | p. 2230.1 by .769. |
| c. .3466 by .0054. | q. .00456 by .1368. |
| d. .001255 by 1004. | r. 12.65 by .01004. |
| e. 2.568 by 15. | s. 3.1146 by .036. |
| f. 7.1125 by 6.5. | t. .0495 by .03888. |
| g. .0625 by 1.75. | u. 6.1325 by 175. |
| h. 3.1 by .0125. | v. 4.8 by .0032. |
| i. .0077 by .875. | w. 4.96723 by 23.9. |
| j. 31.5 by .126. | x. 5.2 by .32. |
| k. 3217 by .0625. | y. .03217 by 6250. |
| l. 4.63638 by 81.34. | z. 15.4546 by .019. |
| m. .429408 by 59.64. | aa. 2147.04 by .036. |
| n. 12.6 by .0012. | bb. .065341 by .000475. |

5. Divide 1 by .1; by .01; and by .0001.

By means of the decimal notation any number large or small may be divided by any other number large or small. But often the quotients are very, very long; and often in them the figures in the quotients repeat themselves after certain stages in the division.

6. 175. divided by 8155 may be carried to very, very many places in the quotient. Carry the figures of the quotient to ten decimal places.

$$7. \quad 10 \div 3 = 3.333333, \text{ etc.} \qquad 10 \div 6 = 1.666666, \text{ etc.}$$

The $3.333 +$ may be indicated by a dot over $\bar{3}$.

The $1.666 +$ may be indicated by a dot over $1.\bar{6}$.

8. Divide 1. by 13; by 17; and by 9.

9. The area of a parallelogram is 8295 sq. ft. and its base is 905.15 ft. What is its altitude?

REVIEW

1. How many coats can be made from 34 yd. of cloth, allowing 4.25 yd. for each coat?

2. How many cubic feet of air are there in a school-room 18 ft. $\times$ 13 ft. 4 in. $\times$ 10 ft. 6 in.

3. At \$25.125 per T., what will 127.045 T. of salt cost?

4. How many cords are there in a pile of cordwood 64 ft. long by 6 ft. high?

5. At \$0.125 per pound, how many pounds of prunes can be bought for \$2.25?

6. How many cubic yards are there in a rectangular excavation for a cellar 24 ft. $\times$ 18 ft. $\times$ 5 ft.?

7. A dealer bought 420.25 tons of potash for \$35.08 a ton. He sold 196.175 tons for \$37.60 a ton, and the rest for \$36.125 a ton. How much did he make?

8. How long will it take a railroad train, traveling 26.18 miles an hour, to travel 366.52 miles?

9. How many tons of soft coal can be put in a rectangular bin 7 ft. long, 4 ft. wide, and 3 ft. deep?

(1 ton of soft coal contains 42 cu. ft.)

10. What will 27.5 bu. of potatoes cost at \$.875 per bu.?

11. A farmer had 23.478 A. in one field, 29.38 A. in a second field, 18.076 A. in a third field, 0.875 A. as garden, and 1.305 A. in an orchard. The rest of his farm of 100 A. was in woodland. How many A. of woodland had he?

12. A man having nine tenths of an acre of land, sold nineteen thousandths of an acre. How much was left?

13. How many tons of hard coal will a rectangular bin 9 ft. long, 5 ft. 6 in. wide, and 4 ft. deep hold?

(1 ton of hard coal contains 33 cu. ft.)

VARIATIONS OF PROBLEMS

I. A boy raised 40 hens from the chickens hatched by 4 hens sitting on an average of 13 eggs each. He paid an average of 2¢ each for the eggs; and for food for all he spent \$4; he received an average of 50¢ each for the 40 hens. What was his gain?

Substitute

A	B	C	D
for 40 hens	for 13 eggs	for 2¢ each	for 50¢ each
1. 35	14	3¢	60¢
2. 30	15	1¢	25¢
3. 25	12	4¢	75¢
4. 20	11	5¢	\$1
5. 50	16	10¢	\$3

6. Write the problem, using this combination of facts:

1 A, 2 B, 3 C, 4 D.

7. Write it, using this combination: 3 A, 4 B, 5 C, 2 D.

II. A girl helped her aunt make 12 mince pies. Her aunt sold them for 35¢ each, and gave her niece $\frac{1}{7}$ of the price for each pie as her share. How much did the girl receive?

	E	F	G
	Substitute for 12 pies	for 35¢	for $\frac{1}{7}$
1.	15	25¢	$\frac{1}{5}$
2.	10	50¢	$\frac{1}{10}$
3.	18	40¢	$\frac{1}{8}$
4.	24	20¢	$\frac{1}{4}$
5.	36	30¢	$\frac{1}{6}$

REVIEW

1. What is the ratio of 18 cu. ft. to 3 cu. yd. ?
2. There are 5.5 yards in one rod, and 1760 yards in a mile. How many rods are there in a mile ?
3. How many feet are there in a mile ?
4. *a.* $159984 \div 144 = ?$ *b.* $33768 \div 168 = ?$
c. $58420 \div 135 = ?$ *d.* $49102 \div 81 = ?$
e. $687310 \div 132 = ?$ *f.* $47631 \div 144 = ?$
g. $876312 \div 240 = ?$ *h.* $76543 \div 108 = ?$
5. What is the ratio of a foot to a mile ?
6. At \$0.21 a pound, how many pounds of Swiss cheese can be bought for \$1613.22 ?
7. *a.* Find the circumference of a circle whose diameter is 6.8 ft. The circumference = diameter $\times$ 3.1416.
b. Find the diameter of a circle whose circumference is 20 in. The diameter of a circle = circumference $\div$ 3.1416.
8. How much wire will be required to make a hoop 2 ft. 6 in. in diameter, allowing 2 in. for the joining ?
9. How many times would such a hoop turn in $\frac{1}{2}$ mi. ?
10. How many tons are there in 3 cars of coal, the first car containing 18.375 tons, the second car 41.625 tons, and the third car 35.5 tons ?
11. If a man earns \$180 in a month of 25 da., how much does he earn in 1 da. ?
12. How long would it take to travel 30.2375 miles at the rate of 8.85 miles per hour ?
13. The prices of 3 kinds of lace are \$2.22 a yard, \$0.98 a yard, and \$1.16 a yard. How many yards of each kind can one buy for \$242.97, if all the money is spent for that one kind ?
14. What is the cost of $\frac{1}{2}$ lb. of bird seed at 4¢ per oz. ?

15. How many ounces are there in 2 lb. of sugar? in 2 lb. of silver?

16. What does the decimal point tell us?

17. If pepper is 40¢ per lb., what will 4 oz. cost?

18. Washington was born 1732 A.D. and lived 67 years. In what year did he die?

19. Babylon is said to have been built in the year 3100 B.C. How long was it to the settlement of Jamestown, Va., 1607 A.D.?

20. A republic was founded in ancient Greece in the year 510 B.C. How long was it from that year to the Declaration of American Independence in 1776 A.D.?

21. X sold a load of wood for \$18. He spent \$4 for groceries and \$6 for meat. How many dollars had he left?

22. A grocer bought 150 dozen eggs, and sold 125 dozen. How many eggs were there left?

23. How many are 29 less 11? 29 less 13? 19 less 5? 19 less 17? 29 less 9? 29 less 16?

24. I bought a bicycle for \$45, and sold it for \$29. How much did I gain or lose?

25. I sold a watch for \$62, which cost me \$85. How much did I gain or lose?

26. I walked 27 miles in 1 day and my brother walked 39 miles. How much farther did he walk?

27. A man had \$110 and paid \$35 to one person and \$40 to another person. How many dollars had he left?

28. How many are $80 - 9 - 6 - 7 - 1 - 2 - 4 - 5 - 6$?

29. How many are $9 + 3 + 4 + 9 + 7 + 6 - 7 - 4 - 3 - 7 - 9 - 1$?

30. William had 67¢; he spent 5¢ for oranges and 25¢ for a flag. How many cents had he left?

DRY MEASURE

The standard unit of *dry measure* is the bushel.

It is $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2150.42 cu. in.

The liquid quart contains $57\frac{3}{4}$ cu. in. and the dry quart $67\frac{1}{2}$ cu. in. What is the difference?

REVIEW

Add :

1.	2.	3.	4.
562347	347326	876208	6752
85476	587765	343672	784519
36728	768246	394625	22687
6544	87335	4713	456275
97328	654	29875	44824
<u>475564</u>	<u>363578</u>	<u>368207</u>	<u>. 612785</u>

5. How many days would a man take to walk 156 miles at the rate of 3 miles an hour for 8 hours a day?

6. Which would fill the larger space, 20 qt. of milk or 20 qt. of corn?

7. How far would a steamer go in 9 days, of 24 hours each, at the rate of 15 miles an hour?

8. $7.25 + 608.125 + 69.785 + 25600.5 + .506 + .00525 = ?$

9. $39.5 + 10.25 + 689.75 + 415.5 + .00075 + 6085.5 = ?$

10. $161700 \div 15 = ?$ $585900 \div 27 = ?$ $3180074 \div 56 = ?$

11. $178464 \div 16 = ?$ $385524 \div 36 = ?$ $9516783 \div 64 = ?$

12. A man bought two farms, one containing 175 acres at \$28 per acre, and the other containing 320 acres at \$37 per acre. What was the cost of both farms?

13. A dealer bought 127 books at 34¢ each, and 97 books at 47¢ each, and sold all of them at 40¢ each. How many cents did he gain?

WRITTEN AND ORAL QUESTIONS

Many problems may be made oral by using sufficiently easy numbers. Compare the following :

1. *a.* How many cord feet are there in 17 cd. of wood ?
1. *b.* ORAL. How many cord feet are there in 5 cd. ?
2. *a.* Mary had 85 % in reading, 90 % in arithmetic, 75 % in nature-study, and 80 % in drawing. If reading and arithmetic each count 2 points, and nature-study and drawing each 1 point, what was her average mark ?
2. *b.* ORAL. Mary had 9 in reading, 9 in arithmetic, 8 in nature-study, and 8 in drawing. If the first two studies each counted two points, and the second two each one point, what was her average ?
3. *a.* Find x . $x : 1250 = 10 : 50$.
3. *b.* ORAL. Find x . $x : 100 = 8 : 16$.
4. *a.* What per cent of 31,000 bushels is 425 bushels ?
4. *b.* ORAL. What per cent of 25 is 2.5 ?
5. A cubic foot of water weighs 62.5 lb. What is the weight of water in a tank 10 ft. $\times$ 5 ft. $\times$ 2 ft. ?
6. A hand is 4 in. What is the height in ft. and in. of a race horse measuring $16\frac{1}{2}$ hands over the front feet ?
7. There are 7.5 gal. in 1 cu. ft. A tank measures 3 ft. $\times$ 5 ft. $\times$ 20 ft. How many gal. does it hold ?
8. A barrel holds 4.2 cu. ft. How many bbl. are needed to hold 158 gal. of water ?
9. How many more cu. in. are in 1 bu. than in 1 cu. ft. ?
10. One horse ate 10 bu. of oats in a month ; another ate 7 bu. of corn. Which ate the greater weight of grain ?
1 bu. of oats weighs 32 lb. 1 bu. of corn 56 lb.
11. How many quarts per day was this for each ?

COMMON FRACTIONS

The number which shows into how many equal parts the unit has been divided is called the **denominator** of the fraction; and the number which shows how many times one of these parts has been taken is called the **numerator** of the fraction. The numerator and the denominator are called the **terms** of the fraction.

We express a fraction in figures by writing the numerator above the denominator and drawing a bar between them; the fraction four fifths, which has 4 for its numerator and 5 for its denominator, is written $\frac{4}{5}$; eleven nineteenth is written $\frac{11}{19}$; thirty-six forty-eighths is written $\frac{36}{48}$.

A fraction expresses an unperformed division, in which the numerator is the dividend and the denominator is the divisor.

A **proper fraction** indicates a quantity less than one whole thing. The numerator of a *proper* fraction is always smaller than the denominator.

$\frac{3}{8}$, $\frac{5}{9}$, $\frac{23}{24}$, $\frac{99}{100}$, are proper fractions. Why?

1. Give other proper fractions.

An **improper fraction** is one whose numerator is not less than its denominator. The value of an *improper* fraction is never less than a unit.

$\frac{6}{5}$, $\frac{13}{12}$, $\frac{100}{24}$, are improper fractions. Why?

2. Give other improper fractions.

A **mixed number** is an integer and a fraction united.

$12\frac{7}{8}$, $39\frac{3}{17}$, $105\frac{7}{12}$, are mixed numbers. Why?

3. Give others, using facts in weights and measures.

A **simple fraction** is one whose numerator and denominator are each single integral numbers.

$\frac{3}{4}$, $\frac{10}{7}$, are simple fractions.

4. Give other simple fractions.

PRINCIPLES OF FRACTIONS

The **value** of the fraction is the quotient obtained by dividing the numerator by the denominator.

$$\text{I. } \frac{1}{3} \times 2 = \frac{2}{3} \qquad \frac{3}{5} \times \frac{1}{5} = \frac{1^2}{5^2} \qquad \frac{7}{50} \times 6 = \frac{42}{50}$$

Multiplying the numerator of a fraction by any number multiplies the fraction by that number.

$$\text{II. } \frac{1}{2 \times 4} = \frac{1}{8} \qquad \frac{1}{5 \times 6} = \frac{1}{30} \qquad \frac{7}{50 \times 2} = \frac{7}{100}$$

Multiplying the denominator of a fraction by any number divides the fraction by that number.

$$\text{III. } \frac{1 \times 2}{2 \times 2} = \frac{2}{4} = \frac{1}{2} \qquad \frac{2}{5} \times \frac{5}{5} = \frac{10}{25} = \frac{2}{5}$$

Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction.

$$\text{IV. } \frac{2 \div 2}{5} = \frac{1}{5} \qquad \frac{10 \div 5}{25} = \frac{2}{5} \qquad \frac{39 \div 13}{50} = \frac{3}{50}$$

Dividing the numerator of a fraction by any number divides the fraction by that number.

$$\text{V. } \frac{10}{25 \div 5} = \frac{10}{5} \qquad \frac{7}{50 \div 5} = \frac{7}{10} \qquad \frac{8}{100 \div 10} = \frac{8}{10}$$

Dividing the denominator of a fraction by any number multiplies the fraction by that number.

$$\text{VI. } \frac{10 \div 5}{25 \div 5} = \frac{2}{5} = \frac{10}{25} \qquad \frac{24 \div 4}{100 \div 4} = \frac{6}{25} = \frac{24}{100}$$

Dividing both numerator and denominator of a fraction by the same number does not affect its value.

$$\text{VII. } \frac{3}{5+1} = \frac{3}{6} \qquad \frac{3}{5-1} = \frac{3}{4} \qquad \frac{3+1}{5} = \frac{4}{5} \qquad \frac{3-1}{5} = \frac{2}{5}$$

Increasing the numerator produces an increase in the value of the fraction; but increasing the denominator decreases its value. Decreasing the numerator decreases the value of the fraction; but decreasing the denominator increases its value.

ANALYSIS OF FRACTIONS

1. In the fraction $\frac{6}{11}$ 11 is the denominator, and shows into how many parts the magnitude has been divided. The parts are elevenths. 6 is the numerator and shows how many parts have been taken. The numbers 6 and 11 are the terms of the fraction. The fraction $\frac{6}{11}$ is an expression of unperformed division, 6 being the dividend and 11 the divisor. It is read six elevenths, and represents 6 of 11 equal parts of 1 or $\frac{1}{11}$ of 6.

2. Analyze the following fractions :

a. $\frac{2}{9}$.	b. $\frac{5}{7}$.	c. $\frac{8}{12}$.	d. $\frac{6}{10}$.	e. $\frac{7}{8}$.	f. $\frac{14}{15}$.
g. $\frac{20}{21}$.	h. $\frac{32}{30}$.	i. $\frac{80}{80}$.	j. $\frac{44}{45}$.	k. $\frac{90}{91}$.	l. $\frac{25}{24}$.
m. $\frac{66}{66}$.	n. $\frac{212}{213}$.	o. $\frac{213}{212}$.	p. $\frac{13}{141}$.	q. $\frac{27}{20}$.	r. $\frac{48}{31}$.
s. $\frac{89}{88}$.	t. $\frac{120}{10}$.	u. $\frac{224}{220}$.	v. $\frac{300}{100}$.	w. $\frac{85}{200}$.	x. $\frac{775}{1000}$.

3. Express in figures :

- a. Three fourths. b. Eight ninths. c. Five sixths.
 d. Seven twelfths. e. Twelve seventeenths.
 f. Four twenty-firsts. g. Eleven forty-fourths.
 h. Thirty-three thirty-fifths. i. Seventy-one eighty-firsts.
 j. One hundred twelve ninety-fifths.
 k. Two hundred five hundredths.

A **complex fraction** is a fraction containing a fraction or a mixed number in either numerator or denominator, or in both numerator and denominator.

$$\frac{\frac{2}{3}}{\frac{7}{8}}, \quad \frac{3}{5\frac{1}{2}}, \quad \frac{2\frac{1}{2}}{4\frac{3}{4}}, \quad \frac{3\frac{1}{2} - 2\frac{2}{3}}{3\frac{1}{2} + 2\frac{2}{3}}$$

A **compound fraction** is a fraction of a fraction :

$$\frac{3}{5} \text{ of } \frac{7}{8}; \quad \frac{13}{15} \times \frac{18}{21}; \quad \frac{6}{7} \div \frac{14}{30}.$$

Both complex and compound fractions may be reduced to simple fractions by division and cancellation, as we shall see later.

REDUCTION OF FRACTIONS

Reduction of fractions changes their forms without altering their values. We may reduce whole numbers to fractions of the same values.

1. Reduce \$5 to quarters.

$$\begin{array}{r} 4 \text{ quarters} \\ 5 \\ \hline 20 \text{ quarters} \end{array}$$
 Since in \$1 there are 4 quarters,
 in \$5 there are 5 times 4 quarters =
 20 quarters.
 or $\frac{5}{1} \times \frac{4}{4} = \frac{20}{4}$.

2. Change $\frac{7}{9}$ to 18ths.

As the denominator is to be 18, and since $18 = 9 \times 2$,
 $\frac{7}{9} \times \frac{2}{2} = \frac{14}{18}$. we multiply both terms by 2. $\frac{14}{18}$ indicates
 equal parts half as large as those in $\frac{7}{9}$, but
 in $\frac{14}{18}$ twice as many parts are taken as in $\frac{7}{9}$.

3. Change:

- a. $\frac{8}{13}$ to 65ths. c. $\frac{11}{15}$ to 90ths. e. $\frac{9}{17}$ to 136ths.
 b. $\frac{5}{6}$ to 72ds. d. $\frac{4}{9}$ to 81sts. f. $\frac{3}{7}$ to 63ds.

QUESTIONS

1. Write five proper fractions; five improper fractions; five mixed numbers.

2. Write five simple fractions; five complex fractions; five compound fractions.

3. Which is the larger amount:

$\frac{5}{6}$ of anything or $\frac{5}{24}$ of it? $\frac{9}{90}$ or $\frac{90}{100}$ of anything?
 $\frac{1}{2}$ of anything or $\frac{1}{16}$ of it? $\frac{7}{8}$ pk. or $\frac{1}{16}$ pk.?

Give the reason in each case, or show by drawing.

4. Which is larger: $\frac{1}{2}$ of $\frac{1}{4}$ or $\frac{1}{4} \times 2$? By how much?

REDUCING INTEGERS TO FRACTIONS

1. Reduce 123 to a fraction with 20 as its denominator.

$$123 = \frac{123}{1}, \quad \frac{123}{1} \times \frac{20}{20} = \frac{2460}{20}$$

2. Change 17 to a fraction with 29 as its denominator.

$$17 = \frac{17}{1}, \quad \frac{17}{1} \times \frac{29}{29} = \frac{493}{29}$$

3. Reduce 7, 9, 27, and 40 to elevenths.

4. Reduce 2, 207, 440, and 9 to fractions with 109 as denominator.

5. Reduce 22, 47, 69, and 100 to ninety-thirds.

6. Reduce 217, 613, 927, and 4 to thirteenths.

REDUCING MIXED NUMBERS TO FRACTIONS

7. Reduce
- $7\frac{4}{9}$
- to an improper fraction.

Since in 1 there are 9 ninths, in 7 there are 7 times 9 ninths. $7 \times \frac{9}{9} = \frac{63}{9}$, $\frac{63}{9} + \frac{4}{9} = \frac{67}{9}$.

8. Change
- $167\frac{14}{19}$
- to an improper fraction.

We multiply the integer by the number in the denominator of the fraction, and write the product as the numerator of an improper fraction. To this improper fraction we add the fraction in the mixed number.

$$167 \times 19 = 3173$$

$$\frac{3173}{19} + \frac{14}{19} = \frac{3187}{19}$$

$$167\frac{14}{19} = \frac{3187}{19}$$

9. Reduce to improper fractions :

<i>a.</i> $73\frac{4}{9}$.	<i>f.</i> $6\frac{4}{13}$.	<i>k.</i> $\$4\frac{9}{10}$.	<i>p.</i> $5\frac{75}{280}$ mi.
<i>b.</i> $18\frac{4}{11}$.	<i>g.</i> $7\frac{3}{8}$.	<i>l.</i> $18\frac{3}{8}$ pk.	<i>q.</i> $82\frac{1}{16}$ hr.
<i>c.</i> $16\frac{3}{7}$.	<i>h.</i> $9\frac{3}{8}$.	<i>m.</i> $10\frac{3}{4}$ cd.	<i>r.</i> $1\frac{800}{1728}$ cu. ft.
<i>d.</i> $14\frac{3}{10}$.	<i>i.</i> $16\frac{3}{8}$.	<i>n.</i> $18\frac{18}{20}$ sq. ft.	<i>s.</i> $36\frac{5}{4}$ da.
<i>e.</i> $71\frac{3}{5}$.	<i>j.</i> $25\frac{1}{2}$.	<i>o.</i> $160\frac{4}{5}$ A.	<i>t.</i> $10\frac{53}{144}$ G. gr.

IMPROPER FRACTIONS

1. Reduce 29 to a fraction having 12 for its denominator.
2. Reduce 243 to a fraction having 3 for its denominator.
3. Reduce 7, 23, and 101 to fractions having 18 for denominator.

4. Reduce to improper fractions :

- a. $39\frac{7}{8}$. d. $676\frac{37}{51}$. g. $847\frac{36}{175}$. j. $375\frac{3}{4}$. m. $1789\frac{5}{9}$.
 b. $112\frac{9}{10}$. e. $464\frac{13}{8}$. h. $675\frac{187}{200}$. k. $375\frac{34}{99}$. n. $125\frac{8}{7}$.
 c. $427\frac{11}{24}$. f. $367\frac{9}{104}$. i. $149\frac{5}{9}$. l. $4834\frac{57}{95}$. o. $10\frac{75}{100}$.

MIXED NUMBERS

5. Reduce $11\frac{1}{5}$ to a mixed number.

$$11\frac{1}{5} = 117 \div 5 = 23\frac{2}{5}$$

$$\begin{array}{r} 5 \overline{)117} \\ \underline{23} \\ 23\frac{2}{5} \end{array}$$

Since 5 fifths = 1, 117
fifths equal as many 1's as
5 is contained times in 117.
5 is found in 117 $23\frac{2}{5}$ times.

6. Change $64\frac{8}{5}$ to a mixed number.

$$64\frac{8}{5} = 648 \div 45 = 14\frac{18}{5} = 14\frac{2}{5}$$

We divide the numer-
ator by the denominator.

7. Reduce to whole or mixed numbers :

- a. $7\frac{1}{4}$. f. $20\frac{0}{7}$. k. $454\frac{7}{8}$. p. $28\frac{4}{32}$. u. $9\frac{6}{8}$.
 b. $169\frac{4}{11}$. g. $17\frac{9}{87}$. l. $12\frac{3}{13}$. q. $396\frac{4}{25}$. v. $100\frac{0}{7}$.
 c. $179\frac{5}{17}$. h. $146\frac{2}{95}$. m. $27\frac{9}{60}$. r. $419\frac{8}{7}$. w. $74\frac{3}{81}$.
 d. $626\frac{2}{44}$. i. $127\frac{9}{125}$. n. $40\frac{7}{13}$. s. $2981\frac{7}{365}$. x. $80\frac{5}{5}$.
 e. $71\frac{4}{19}$. j. $11\frac{3}{17}$. o. $60\frac{0}{25}$. t. $53\frac{9}{89}$. y. $172\frac{8}{12}$.

8. Reduce to mixed numbers :

- a. $18\frac{0}{9}$ bu. b. $29\frac{0}{9}$ sq. ft. c. $3000\frac{0}{640}$ A. d. $531\frac{0}{1}$ cu. in.
 e. $18\frac{3}{6}$ doz. f. $12\frac{9}{8}$ cd. g. $7\frac{5}{4}$ gal. h. $1000\frac{0}{8}$ rd.

REDUCTION OF DECIMALS TO COMMON FRACTIONS

1. Reduce .95 to a common fraction at its lowest terms.

$$.95 = \frac{95}{100} = \frac{19}{20}$$

We divide 95 and 100 by the common factor 5.

2. Reduce .0125 to a common fraction at its lowest terms.

$$.0125 = \frac{125}{10000} = \frac{25}{2000} = \frac{5}{400} = \frac{1}{80}$$

We divide 125 and 10000 by 5, 5, and 5, the common factors of the numerator and denominator.

3. Reduce to a common fraction at lowest terms:

<i>a.</i> .0025.	<i>d.</i> .204.	<i>g.</i> .435.	<i>j.</i> .124.
<i>b.</i> .0065.	<i>e.</i> .05.	<i>h.</i> .575.	<i>k.</i> .486.
<i>c.</i> .375.	<i>f.</i> .225.	<i>i.</i> .012.	<i>l.</i> .225.

It is very seldom helpful to change a decimal to a common fraction with a denominator larger than one hundred. It is more often helpful to change a common fraction to a decimal than to change a decimal to a common fraction.

RECITE

REVIEW

4. What is the ratio of 1 to .25? of .25 to 1?
5. What is the ratio of 1 to .3? of .3 to 1?
6. What is the ratio of 1.25 to 10? of 10 to 1.25?
7. What is the difference between

<i>a.</i> \$575 and \$.50?	<i>b.</i> \$3.75 and \$1.375?
<i>c.</i> \$1 and \$.66+?	<i>d.</i> \$2.50 and \$1.166+?
8. What is the sum of \$12.33+ and \$12.66+?
9. Make drawings to illustrate 4, 5, and 6.

REDUCTION OF COMMON FRACTIONS TO DECIMALS

1. Change $\frac{3}{5}$ into a decimal.

$$\begin{array}{r} 5 \overline{)3.0} \\ \underline{.6} \end{array}$$

**We divide the numerator
by the denominator.**

There is one decimal place in the dividend, and none in the divisor; therefore, there is one decimal place in the quotient.

2. Reduce $\frac{5}{16}$ to a decimal.

$$\begin{array}{r} 16 \overline{)5.0000(.3125} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \end{array}$$

The numerator is treated as a whole number; any annexed ciphers are treated as indicating decimal fraction places.

We point off the quotient as always in the division of decimals.

3. Change to decimals:

a. $\frac{1}{2}$.

c. $\frac{17}{60}$.

e. $\frac{5}{8}$.

g. $\frac{7}{64}$.

i. $\frac{1}{75}$.

b. $\frac{4}{5}$.

d. $\frac{3}{4}$.

f. $\frac{5}{16}$.

h. $\frac{3}{50}$.

j. $\frac{84}{138}$.

4. Change $\frac{2}{3}$ to a decimal.

$$\begin{array}{r} 3 \overline{)2.00} \\ \underline{.66\frac{2}{3}} \end{array}$$

We may write the answer
.66+ or . $\dot{6}$ or $.6\frac{2}{3}$.

A common fraction whose denominator contains any factor other than 2 or 5 cannot be reduced to a pure decimal without remainder, unless its own numerator contains that other factor. The decimals with remainders in the common fraction form, or of that value, are "approximate" decimals.

ADDITIONS

	A	B	C	D	E	F	G	H	I
J	1	2	3	4	5	6	7	8	9
K	2	3	4	5	6	7	8	9	1
L	1	9	3	4	7	6	5	8	2
M	3	4	5	6	1	8	9	7	2
N	1	2	3	4	5	9	9	6	8
O	4	5	6	7	8	9	9	5	1
P	2	1	3	4	5	8	8	5	7
Q	5	6	7	8	9	7	8	5	3
R	1	2	3	4	5	6	7	4	8
S	6	7	8	9	5	6	8	5	3
T	1	3	2	4	4	5	8	5	6
U	7	8	9	4	3	4	8	4	2
V	1	2	3	3	2	3	7	4	5
W	8	9	2	4	1	3	7	4	5
X	1	2	2	5	2	3	7	3	4
Y	9	1	3	6	4	1	6	2	8
Z	1	1	3	7	4	6	5	1	2
	—	—	—	—	—	—	—	—	—

Prove by adding from both top and bottom or from both left and right.

1. Add by columns. 2. Add by rows.
3. Add diagonally : from AJ to IR; from IR to AZ.
4. Add from left to right and from right to left.
5. Note easy combinations. $5 + 5 = 10$, $4 + 4 + 2 = 10$, $7 + 7 + 7 = 21$, $8 + 7 = 15$, $4 + 1 + 3 + 7 = 15$, etc.
6. Add orally in recitation; and in writing.
7. Add two, three, or more columns written together :

12	23	34	567	6789
23,	34,	45,	678,	7891, etc.

REVIEW

1. What is the least common multiple of 4, 6, 8?
2. What is the greatest common divisor of 4, 6, 8?
3. What are the prime factors of 6, 8, 9, 10, 12?
4. If the units' figure of a number may be divided by 2, what do we know is one factor of the number?
5. If the sum of the digits of a number may be divided by 3, what do we know of the factors of a number?
6. If the final digit is 0 or 5, what do we know of the number?
7. What is a prime factor?
8. Give some numbers that have to each other the ratios of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.
9. Reduce to improper fractions: $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $5\frac{2}{5}$, $6\frac{5}{6}$.
10. Reduce to fifths these fractions: $\frac{20}{30}$, $\frac{40}{60}$, $\frac{80}{100}$.
11. What is the volume of a prism $5 \times 5 \times 8$?
12. What is the volume of a prism of which the ratio to the above prism is 5?
13. If the ratio were $\frac{1}{5}$ of the first prism, what would the volume of a third prism be?
14. The size of a garden is 15 ft. $\times$ 20 ft. How many feet of wire netting must the gardener get to protect the garden from fowls and animals?
15. What is the ratio of one million dollars to one billion? of a billion to one million?
16. What per cent is $\frac{1}{5}$ of anything? $\frac{1}{5}$ of it?
17. Which is greater, $\frac{2}{3}$ or 70% of anything? By how much?

LOWEST TERMS

A fraction is in its lowest terms when its terms are prime to each other. What are the *terms* of a fraction?

1. Reduce $\frac{12}{36}$ to its lowest terms.

$\frac{12}{36} = \frac{1}{3}$ We divide the terms of the fraction by 12, a factor common to them both, and obtain $\frac{1}{3}$.

Since 1 and 3 are prime to each other, the fraction $\frac{1}{3}$ is in its lowest terms.

Since numerator and denominator correspond to dividend and divisor, dividing both by the same number changes only the form of the fraction, the value of the fraction remaining the same.

2. Reduce $\frac{315}{1890}$ to its lowest terms.

2 will not divide 315, since 315 is not an even number.

315 and 1890 divided by 3 give this new fraction :

We cancel out the 3.

105 and 630 divided by 5 give this new fraction :

We cancel out the 5.

21 and 126 divided by 3 give :

We cancel out another 3.

Last, we cancel out the 7.

At its lowest terms $\frac{315}{1890} = \frac{1}{6}$.

When the terms of the fraction to be reduced are large, we cancel in each term the common prime factors, until the terms are prime to each other.

3. Reduce to lowest terms :

a. $\frac{12}{15}$. b. $\frac{27}{36}$. c. $\frac{144}{864}$. d. $\frac{288}{2592}$. e. $\frac{275}{440}$. f. $\frac{330}{810}$.

$$\begin{aligned} \frac{315}{1890} &= \frac{105}{630} \\ \frac{105}{630} &= \frac{21}{126} \\ \frac{21}{126} &= \frac{7}{42} \\ \frac{7}{42} &= \frac{1}{6} \end{aligned}$$

4. Reduce to lowest terms :

- a.* $\frac{18}{24}$. *b.* $\frac{24}{36}$. *c.* $\frac{36}{144}$. *d.* $\frac{63}{81}$. *e.* $\frac{4}{6}$. *f.* $\frac{3}{9}$.
g. $\frac{4}{12}$. *h.* $\frac{7}{21}$. *i.* $\frac{24}{48}$. *j.* $\frac{32}{64}$. *k.* $\frac{60}{96}$. *l.* $\frac{12}{144}$.
m. $\frac{9}{99}$. *n.* $\frac{72}{96}$. *o.* $\frac{132}{144}$. *p.* $\frac{42}{63}$. *q.* $\frac{81}{90}$. *r.* $\frac{144}{168}$.

5. Reduce to lowest terms :

WRITE

- a.* $\frac{58760}{66105}$. *b.* $\frac{315}{405}$. *c.* $\frac{303}{506}$. *d.* $\frac{364}{404}$. *e.* $\frac{728}{808}$.
f. $\frac{1362}{1500}$. *g.* $\frac{81}{436}$. *h.* $\frac{168}{216}$. *i.* $\frac{160}{352}$. *j.* $\frac{144}{208}$.
k. $\frac{120}{225}$. *l.* $\frac{784}{1232}$. *m.* $\frac{230400}{537600}$. *n.* $\frac{28}{303}$. *o.* $\frac{1500}{1600}$.
p. $\frac{990}{1000}$. *q.* $\frac{10000}{100000}$. *r.* $\frac{1000}{100000}$. *s.* $\frac{500}{25000}$. *t.* $\frac{110}{10000}$.

REDUCTION TO A COMMON DENOMINATOR

6. Reduce $\frac{2}{3}$, $\frac{7}{11}$, and $\frac{5}{9}$ to a common denominator.

$$3 \times 11 \times 9 = 297 = \text{first numerator}$$

$$7 \times 4 \times 9 = 252 = \text{second numerator}$$

$$5 \times 4 \times 11 = 220 = \text{third numerator}$$

$$4 \times 11 \times 9 = 396 = \text{common denominator}$$

$$\frac{2}{3} = \frac{227}{396}, \quad \frac{7}{11} = \frac{252}{396}, \quad \text{and} \quad \frac{5}{9} = \frac{220}{396}$$

We multiply
the numerator of
each fraction by
the denomina-
tors of the other
fractions to find
the new numer-

ators; and we multiply all the denominators together for the new common denominator.

7. Reduce to equivalent fractions with a common denominator :

- a.* $\frac{1}{2}, \frac{2}{3}, \frac{4}{7}, \frac{9}{11}$. *b.* $\frac{18}{20}, \frac{6}{7}, \frac{4}{5}$. *c.* $\frac{2}{5}, \frac{7}{10}, \frac{4}{9}$. *d.* $\frac{1}{2}, \frac{10}{11}, \frac{1}{3}$.
e. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}$. *f.* $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. *g.* $\frac{2}{3}, \frac{3}{4}, \frac{1}{5}$. *h.* $\frac{1}{3}, \frac{2}{3}, \frac{3}{4}$.
i. $\frac{5}{8}, \frac{7}{9}, \frac{2}{3}$. *j.* $\frac{9}{10}, \frac{10}{11}, \frac{11}{12}$. *k.* $\frac{12}{13}, \frac{13}{14}, \frac{14}{15}$. *l.* $\frac{4}{5}, \frac{5}{6}, \frac{6}{7}$.
m. $\frac{10}{11}, \frac{11}{12}, \frac{12}{13}$. *n.* $\frac{14}{15}, \frac{15}{16}, \frac{16}{17}$. *o.* $\frac{10}{20}, \frac{27}{4}, \frac{12}{30}$. *p.* $\frac{8}{9}, \frac{3}{10}, \frac{9}{12}$.
q. $\frac{6}{12}, \frac{4}{16}, \frac{8}{20}$. *r.* $\frac{5}{7}, \frac{1}{8}, \frac{5}{9}$. *s.* $\frac{7}{8}, \frac{35}{72}, \frac{1}{100}$. *t.* $\frac{1}{4}, \frac{1}{10}, \frac{1}{75}$.

LEAST COMMON DENOMINATOR

The **least common denominator** of two or more fractions is the least denominator to which they all can be reduced. The least common denominator must be the least common multiple of the denominators.

1. Reduce $\frac{5}{6}$, $\frac{8}{9}$, and $\frac{1}{12}$ to their least common denominator.

$$\begin{array}{r|rrrr} 3 & 6 & 9 & 12 & \\ 2 & 2 & 3 & 4 & \\ \hline & 1 & 3 & 2 & \end{array}$$

$$3 \times 2 \times 3 \times 2 = 36$$

$$6 \overline{)36}$$

$$\frac{6}{6}$$

$$5 \times 6 = 30 \quad \frac{5}{6} = \frac{30}{36}$$

$$9 \overline{)36}$$

$$\frac{4}{9}$$

$$8 \times 4 = 32 \quad \frac{8}{9} = \frac{32}{36}$$

$$12 \overline{)36}$$

$$\frac{3}{12}$$

$$1 \times 3 = 3 \quad \frac{1}{12} = \frac{3}{36}$$

We find the least common multiple of the denominators, and divide it by the denominator of each fraction. Each quotient multiplied by the numerator of each respective fraction gives the new numerator for the fraction.

2. Reduce $\frac{1}{4}$, $\frac{7}{12}$, $\frac{2}{3}$, and $\frac{9}{16}$ to their least common denominator.

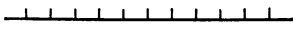
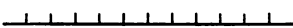
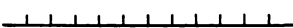
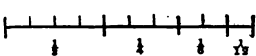
3. Reduce to their least common denominators:

Each fraction should first be reduced to its lowest terms, and mixed numbers should be changed to improper fractions.

- a.* $\frac{4}{5}, \frac{8}{9}, \frac{3}{15}$. *f.* $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$. *k.* $\frac{2}{5}, \frac{4}{6}, \frac{5}{9}, \frac{7}{10}$. *p.* $\frac{3}{4}, \frac{5}{6}, \frac{2}{5}, \frac{3}{8}, \frac{5}{10}$.
b. $\frac{5}{6}, \frac{4}{9}, \frac{7}{12}$. *g.* $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}$. *l.* $\frac{67}{120}, \frac{4}{5}, \frac{5}{6}$. *q.* $3\frac{1}{2}, 4\frac{1}{3}, \frac{6}{5}, \frac{9}{15}$.
c. $\frac{4}{5}, 3\frac{6}{20}$. *h.* $\frac{7}{8}, \frac{9}{10}, 7\frac{3}{4}$. *m.* $\frac{5}{12}, \frac{7}{15}, \frac{11}{24}$. *r.* $\frac{3}{7}, \frac{9}{14}, \frac{11}{28}, 5\frac{3}{7}$.
d. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$. *i.* $1\frac{1}{2}, \frac{2}{3}, 3\frac{1}{4}$. *n.* $\frac{1}{10}, \frac{1}{3}, \frac{1}{7}, \frac{4}{21}$. *s.* $\frac{2}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{3}{15}$.
e. $\frac{3}{15}, \frac{2}{24}, \frac{8}{9}$. *j.* $\frac{1}{24}, \frac{1}{36}, \frac{3}{48}$. *o.* $3\frac{1}{8}, 4\frac{4}{12}, 8\frac{6}{18}$. *t.* $\frac{7}{10}, \frac{13}{40}, \frac{29}{80}, \frac{57}{160}$.
u. $\frac{2}{30}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{9}{20}$. *v.* $\frac{2}{3}, \frac{1}{4}, \frac{3}{5}, \frac{5}{6}, \frac{9}{10}, \frac{7}{120}$. *w.* $\frac{4}{5}, \frac{3}{8}, \frac{4}{6}, \frac{3}{4}, \frac{7}{15}$.
x. $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \frac{13}{15}, \frac{7}{16}$. *y.* $\frac{4}{5}, \frac{6}{11}, \frac{29}{20}, \frac{31}{44}, \frac{19}{55}, \frac{3}{4}$.

ADDITION OF FRACTIONS

Fractions can be added only when they express fractional parts of the same kind.

1. To add $\frac{1}{8}$  of  1,
 $\frac{1}{4}$  of  1,
 $\frac{1}{6}$  of  1,
and $\frac{1}{12}$  of  1,
we must reduce $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$ to fractional parts of the same kind. This gives us  = $\frac{10}{24}$.

We reduce the fractions to a common denominator and add. It is usually best to reduce to the least common denominator.

When we have improper fractions and mixed numbers to add together, it is usually better to reduce the improper fractions to mixed numbers, to add the whole numbers, and then to add the fractions when reduced to a least common denominator.

2. Add $\frac{1}{3} + \frac{2}{5} + \frac{4}{15} + \frac{1}{12}$.
3. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{10}$, $\frac{3}{15}$.
4. Add $2\frac{1}{3}$, $3\frac{2}{5}$, $\frac{4}{15}$, 5.
5. Add $3\frac{1}{6}$, $10\frac{1}{6}$, $10\frac{5}{12}$.
6. What is the sum of $6\frac{2}{7} + 19\frac{4}{11} + 9\frac{3}{8} + 17\frac{6}{7} + 23\frac{13}{14}$?
7. Add $2\frac{1}{3}$, $3\frac{2}{5}$, $\frac{4}{15}$, $5\frac{1}{12}$.
8. Add $2\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{7}{9}$, $6\frac{2}{3}$.
9. Add $3\frac{1}{3}$, $6\frac{1}{6}$, $9\frac{1}{9}$, $10\frac{2}{12}$.
10. Add $1\frac{1}{6}$, $\frac{2}{7}$, $3\frac{1}{4}$, $7\frac{1}{2}$.
11. How many acres are there in 3 lots where the first lot contains $17\frac{3}{8}$ acres, the second $25\frac{2}{3}$ acres, and the third $46\frac{8}{15}$ acres?

12. A man sold $112\frac{6}{7}$ bushels of grain for $250\frac{1}{2}$ dollars; $9\frac{5}{12}$ bushels for $62\frac{3}{8}$ dollars; $225\frac{2}{14}$ bushels for $104\frac{1}{6}$ dollars. How many bushels did he sell? How much money did he receive for the grain?

CANCELLATION

One factor in the dividend cancels only one equal factor in the divisor.

1. Multiply 210 by 36 and divide the product by 60.

$$\frac{210 \times 36}{60} = \frac{42 \times 3}{1} = \frac{126}{1} = 126$$

$\begin{array}{r} 42 \quad 3 \\ \cancel{210} \times \cancel{36} \\ \hline 60 \\ \hline 6 \\ \hline 1 \end{array}$

We remove the factor 12 from the 36 in the dividend, and from the 60 in the divisor.

Dividing by 5 the quotient, 5, in the divisor and the factor 210 in the dividend, we have left in the dividend the factors 42 and 3. Their product is 126. 1 is the only factor left in the divisor. The quotient is $\frac{126}{1} = 126$.

2. $\frac{10 \times 15 \times 16}{25 \times 24} = ?$

5. $\frac{18 \times 20 \times 21}{28 \times 30} = ?$

3. $\frac{27 \times 28 \times 30}{35 \times 36 \times 18} = ?$

6. $\frac{27 \times 32 \times 36}{21 \times 36 \times 40} = ?$

4. $\frac{54 \times 55 \times 56}{35 \times 66 \times 72} = ?$

7. $\frac{55 \times 56 \times 60}{25 \times 32 \times 77} = ?$

8. What is the value of $36 \times 42 \times 52$ divided by $2 \times 3 \times 4$? divided by 9×21 ? divided by 21×13 ?

Tell the answers without doing any written work :

9. $\frac{2 \times 3 \times 9}{27} = ?$

13. $\frac{5 \times 5}{50} = ?$

17. $\frac{20}{2 \times 4 \times 10} = ?$

10. $\frac{3 \times 5 \times 7}{21} = ?$

14. $\frac{5 \times 6 \times 8}{60} = ?$

18. $\frac{4 \times 8}{2 \times 16} = ?$

11. $\frac{4 \times 4 \times 5}{40} = ?$

15. $\frac{18}{2 \times 3 \times 3} = ?$

19. $\frac{6 \times 10}{5 \times 12} = ?$

12. $\frac{4 \times 6 \times 8}{48} = ?$

16. $\frac{320}{4 \times 5 \times 8} = ?$

20. $\frac{8 \times 18}{16 \times 27} = ?$

FACTORING

An **exponent** is a figure written above and at the right of a number, and denotes how many times the latter is repeated as a factor; thus, $3^2 = 3 \times 3$; $7^3 = 7 \times 7 \times 7$; $2^5 = 2 \times 2 \times 2 \times 2 \times 2$.

1. Is 689 a prime number?

689 is not divisible by 2 because its last digit is an odd number, or by 3 or by 7 or by 11, but is divisible by 13; therefore, 689 is not a prime.

2. Is 947 a prime?

947 is not divisible by 2, 3, 7, 11, 13, 17, 19, 23, or 29; and when divided by the next prime, 31, the quotient is less than 31; therefore, 947 is a prime.

3. Find the prime factors of 44,856.

2	44856	The number is divisible by 2, giving the quotient 22,428; 22,428 is divisible by 2, giving the quotient 11,214; 11,214 is divisible by 2, giving the quotient 5607; 5607 is divisible by 3, giving the quotient 1869; 1869 is divisible by 3, giving the quotient 623; but 623 is not divisible by 3.
2	22428	
2	11214	
3	5607	
3	1869	
7	623	
	89	

Nor is 623 divisible by the next prime, 5; but it is divisible by 7, giving the quotient 89; and 89 is not divisible by 7. The square of 11, the next prime number, is 121, a number greater than 89; therefore, 89 must be prime. Hence,

$$44856 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 89 = 2^3 \cdot 3^2 \cdot 7 \cdot 89.$$

4. Find the prime factors of:

- 12, 14, 15, 16, 18, 20, 22, 24, 46, 48, 49, 50, 51, 52.
- 25, 26, 27, 28, 30, 32, 33, 34, 56, 57, 58, 60, 62, 63.
- 35, 36, 38, 39, 40, 42, 44, 45, 54, 55, 64, 65, 66, 68.

5. Find the prime factors of:

- a. 67, 69, 70, 72, 85, 87, 90, 95, 98.
 b. 102, 111, 119, 125, 138, 146, 155.
 c. 154, 166, 178, 209, 234, 259, 267.
 d. 309, 366, 375, 404, 473, 524, 647.
 e. 6018, 76540, 63142, 78900, 6432, 97563, 89706.
 f. 7498, 56234, 49750, 3333, 99939, 48765, 92890.

6. Resolve into their prime factors the numbers 29,656 and 15,725.

$$\begin{array}{r} 8 \overline{) 29656} \\ 11 \overline{) 3707} \\ \quad 337 \end{array}$$
 29,656 is divisible by 8. The quotient is divisible by 11. The quotient is now 337; which is less than $400 = 20 \times 20$. Hence, unless 337 is prime, one of its factors is

less than 20. No number under 12 divides it, and on trial with 13, 17, $= 8 \times 11 \times 337$. and 19 we find that 337 has no factor under 20.

$$\begin{array}{r} 5 \overline{) 15725} \\ 5 \overline{) 3145} \\ 17 \overline{) 629} \\ \quad 37 \end{array}$$
 15,725 can be divided by 5, and again by 5, giving the quotient 629. But 629 is less than 30×30 , and its divisors are therefore less than 30. By trial with 13, 17, since it is divisible by no number under 12, we find

17 is contained 37 times in the number.

Therefore $15,725 = 5 \times 5 \times 17 \times 37 = 5^2 \times 37 \times 17$.

7. Resolve into their prime factors :

- | | | | |
|-----------|-----------|------------|------------|
| a. 1602. | h. 1827. | o. 555. | v. 2934. |
| b. 1632. | i. 36768. | p. 93555. | w. 32320. |
| c. 1365. | j. 851. | q. 30012. | x. 1011. |
| d. 15296. | k. 528. | r. 333333. | y. 707. |
| e. 378. | l. 1485. | s. 1118. | z. 19061. |
| f. 9625. | m. 13237. | t. 11368. | a'. 20625. |
| g. 616. | n. 15296. | u. 1024. | b'. 49560. |

8-9. Which of the following numbers are prime?

Find the prime factors of those which are composite :

a. 37, 4393, 93, 101, 7771, 111, 227.

b. 107, 9959, 113, 110, 1793, 207, 469.

c. 239, 7111, 349, 299, 713, 169, 3029.

d. 507, 6539, 2197, 2327, 581, 4573, 1363.

10. Divide the product of 45 times 84 by 9.

11. $15 \times 7 \times 12$ by $5 \times 3 \times 7 \times 2$.

12. $75 \times 15 \times 24$ by $25 \times 3 \times 6 \times 4 \times 5$.

13. What is the value of $99 \times 360 \times 365$ divided by 11×73 ? divided by 33×18 ? divided by 44×5 ?

14. What is the value of $9 \times 40 \times 100$ divided by $6 \times 8 \times 10$? divided by $3 \times 6 \times 25$?

15. $\frac{22 \times 24 \times 25}{33 \times 40} = ?$

23. $\frac{70 \times 72 \times 80}{56 \times 60 \times 60} = ?$

16. $\frac{33 \times 35 \times 36}{27 \times 28 \times 55} = ?$

24. $\frac{30 \times 32 \times 33}{44 \times 48} = ?$

17. $\frac{56 \times 64 \times 65}{40 \times 32 \times 26} = ?$

25. $\frac{45 \times 48 \times 50}{25 \times 36 \times 60} = ?$

18. $\frac{27 \times 35 \times 40}{45 \times 56} = ?$

26. $\frac{75 \times 80 \times 81}{45 \times 45 \times 60} = ?$

19. $\frac{35 \times 40 \times 42}{25 \times 48 \times 49} = ?$

27. $\frac{28 \times 30 \times 36}{42 \times 45} = ?$

20. $\frac{64 \times 65 \times 70}{50 \times 52 \times 56} = ?$

28. $\frac{54 \times 56 \times 60}{42 \times 45 \times 48} = ?$

21. $\frac{24 \times 28 \times 30}{36 \times 42} = ?$

29. $\frac{80 \times 91 \times 90}{45 \times 56 \times 130} = ?$

22. $\frac{44 \times 45 \times 49}{28 \times 33 \times 35} = ?$

30. $\frac{1000000}{8 \times 40 \div 2 + 5} = ?$

31. 11 doz. suits were sold at \$9.75 a suit in equal lots to 22 different men. How much in all did each pay?

FRACTIONS

1. Add:

a. $\frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{3}{8}, \frac{8}{11}$.

b. $4\frac{1}{4}, 1\frac{1}{3}, \frac{7}{11}$.

c. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}$.

d. $\frac{3}{4}, \frac{5}{6}, \frac{4}{5}$.

e. $16\frac{3}{11}, 47\frac{2}{5}, 21\frac{17}{13}, \frac{7}{18}, 19\frac{1}{2}$.

f. $17\frac{1}{2}, 43\frac{3}{7}, 168\frac{4}{9}, 207\frac{8}{21}, 506\frac{125}{128}$.

2. A grocer sold $47\frac{3}{4}$ pounds of coffee in one day, $83\frac{3}{5}$ pounds the next day, and $68\frac{5}{9}$ pounds the next day. How many pounds did he sell in the 3 days?

3. I traveled $85\frac{5}{12}$ miles in one day, $78\frac{9}{16}$ in another day, and $125\frac{7}{5}$ in another day. How many miles did I travel in the 3 days?

4. A man bought 3 bolts of cloth, one containing $127\frac{7}{8}$ yards, another $168\frac{9}{16}$ yards, and the third $256\frac{3}{5}$ yards. How many yards did he buy?

5. I bought a table for $\$2\frac{5}{8}$, a sofa for $\$9\frac{5}{8}$, and a bookcase for $\$5\frac{3}{4}$. What was the total cost?

6. A grocer bought 4 loads of potatoes containing respectively $54\frac{3}{4}$, $55\frac{3}{8}$, $51\frac{7}{16}$, and $50\frac{21}{32}$ bushels. How many bushels did he buy?

7. I paid for sugar at one time $\$1\frac{7}{12}$, at another time $\$3\frac{3}{8}$, at another $\$7\frac{3}{4}$, and at another time $\$5\frac{1}{6}$. How much did I pay in all?

8. A dealer sold three pieces of silk, the first piece contained $22\frac{5}{6}$ yards, the second $20\frac{7}{8}$ yards, and the third $21\frac{1}{4}$ yards. How many yards were there in the three pieces?

9. I bought 5 tubs of butter; the first tub contained $18\frac{7}{12}$ lb., the second $19\frac{11}{20}$ lb., the third $19\frac{5}{8}$ lb., the fourth $21\frac{11}{5}$ lb., and the fifth $20\frac{13}{18}$ lb. How many pounds of butter were there in the 5 tubs?

REDUCTION OF DECIMALS

1. Change to decimals:

<i>a.</i> $\frac{1}{3}$.	<i>f.</i> $\frac{3}{8}$.	<i>k.</i> $\frac{8}{9}$.	<i>p.</i> $\frac{13}{22}$.	<i>u.</i> $\frac{1}{88}$.
<i>b.</i> $\frac{1}{6}$.	<i>g.</i> $\frac{5}{12}$.	<i>l.</i> $\frac{11}{12}$.	<i>q.</i> $\frac{11}{36}$.	<i>v.</i> $\frac{1}{133}$.
<i>c.</i> $\frac{5}{8}$.	<i>h.</i> $\frac{7}{8}$.	<i>m.</i> $\frac{4}{9}$.	<i>r.</i> $\frac{3}{50}$.	<i>w.</i> $\frac{1}{200}$.
<i>d.</i> $\frac{1}{12}$.	<i>i.</i> $\frac{5}{18}$.	<i>n.</i> $\frac{5}{11}$.	<i>s.</i> $\frac{7}{60}$.	<i>x.</i> $\frac{1}{500}$.
<i>e.</i> $\frac{5}{16}$.	<i>j.</i> $\frac{7}{24}$.	<i>o.</i> $\frac{8}{15}$.	<i>t.</i> $\frac{2}{75}$.	<i>y.</i> $\frac{1}{2500}$.

2. Change to decimals, carrying out to not over five decimal places:

<i>a.</i> $\frac{6}{7}$.	<i>b.</i> $\frac{13}{17}$.	<i>c.</i> $\frac{8}{17}$.	<i>d.</i> $\frac{3}{13}$.	<i>e.</i> $\frac{47}{51}$.
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STUDY TABLE OF EQUIVALENTS

I $\frac{1}{3} = .33+$	IX $\frac{1}{6} = .166+$	Prove these equalities by reducing the common fractions to decimals. Make drawings to show some of the quantities indicated.
II $\frac{2}{3} = .66+$	X $\frac{5}{6} = .833+$	
III $\frac{1}{4} = .25$	XI $\frac{1}{8} = .125$	
IV $\frac{3}{4} = .75$	XII $\frac{3}{8} = .375$	
V $\frac{1}{5} = .2$	XIII $\frac{5}{8} = .625$	
VI $\frac{2}{5} = .4$	XIV $\frac{7}{8} = .875$	
VII $\frac{3}{5} = .6$	XV $\frac{1}{12} = .0833+$	
VIII $\frac{4}{5} = .8$	XVI $\frac{5}{12} = .5833+$	
XVII $\frac{1}{2} = .5 = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$.		
XVIII $\frac{1}{3} = .33+ = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$.		
XIX $\frac{2}{3} = .66+ = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{13}{18}$.		

RECITE

REVIEW

- What is the ratio of 75% of anything to $12\frac{1}{2}\%$ of it?
- What is the ratio of .125 to .375?
- What per cent is 4 of 10? 3 of 10?
- $87\frac{1}{2}\%$ of a thing less 50% of it is how many eighths?

SUBTRACTION OF FRACTIONS

What is meant by *subtraction*? Explain *minuend*, *subtrahend*, *remainder*. How many numbers may be added at one time? How many may be subtracted?

One fraction can be subtracted from another only when both express fractional units of the same kind.

1. Find the difference between $\frac{2}{3}$ and $\frac{5}{7}$.

Why can we neither add nor subtract **unlike** fractions?

$$\frac{2}{3} = \frac{14}{21}, \quad \frac{5}{7} = \frac{15}{21}, \quad \frac{5}{7} - \frac{2}{3} = \frac{15}{21} - \frac{14}{21} = \frac{1}{21}$$

First we reduce the fractions to equivalent fractions having the lowest common denominator. Then we subtract the numerator of the subtrahend from the numerator of the minuend, and write the result over the denominator.

2. Find the value of:

$$\begin{array}{llll} a. \frac{7}{18} - \frac{4}{21}. & b. \frac{19}{20} - \frac{11}{16}. & c. \frac{111}{200} - \frac{1}{10}. & d. \frac{1}{10} - \frac{1}{1000}. \\ e. \frac{17}{24} - \frac{7}{20}. & f. \frac{11}{34} - \frac{1}{10}. & g. \frac{3}{5} - \frac{1}{4}. & h. \frac{12}{15} - \frac{8}{15}. \\ i. \frac{31}{36} - \frac{9}{16}. & j. \frac{18}{37} - \frac{3}{11}. & k. \frac{25}{35} - \frac{17}{35}. & l. \frac{18}{27} - \frac{3}{9}. \end{array}$$

3. From $192\frac{2}{11}$ take $161\frac{5}{16}$.

$\frac{2}{11}$ and $\frac{5}{16}$ reduced to a common denominator become $\frac{32}{176}$ and $\frac{165}{176}$.

$$192\frac{2}{11} - 161\frac{5}{16} = 192\frac{32}{176} - 161\frac{165}{176} = 191 + 1\frac{32}{176} - 161\frac{165}{176} = 191\frac{208}{176} - 161\frac{165}{176} = 175\frac{43}{176}.$$

Since we cannot subtract $\frac{165}{176}$ from $\frac{32}{176}$, we take 1 from the integral part of the minuend, and considering it as $\frac{176}{176}$ add it to $\frac{32}{176}$. $192\frac{32}{176} = 191\frac{208}{176}$.

4. A boy rode in two days on his bicycle $53\frac{1}{10}$ mi. The first day he rode $29\frac{2}{5}$ mi. How far did he ride the second day?

5. $9\frac{1}{18}$ yd. $- 5\frac{7}{12}$ yd. = ?

6. From $29\frac{2}{11}$ take $16\frac{4}{7}$.

$$29\frac{2}{11} - 16\frac{4}{7} = 29\frac{14}{77} - 16\frac{44}{77} = 28 + 1\frac{14}{77} - 16\frac{44}{77} = 28\frac{81}{77} - 16\frac{44}{77} = 12\frac{37}{77}.$$

7. From $117\frac{3}{8}$ take $67\frac{40}{45}$.

8. Find the value of:

a. $12\frac{1}{10} - \frac{36}{47}$. b. $26\frac{3}{4} - 15\frac{8}{3}$. c. $46\frac{5}{16} - 27\frac{4}{19}$. d. $19\frac{3}{8} - 14\frac{7}{11}$.

9. Subtract:

a. $31\frac{1}{4}$	b. $42\frac{4}{7}$	c. $16\frac{6}{16}$	d. $15\frac{3}{8}$	e. $13\frac{3}{7}$
$\underline{6\frac{1}{4}}$	$\underline{18\frac{3}{8}}$	$\underline{5\frac{3}{9}}$	$\underline{\frac{3}{4}}$	$\underline{9\frac{1}{7}}$

f. $62\frac{1}{2}$	g. 38	h. 14	i. 105	j. $25\frac{4}{5}$
$\underline{\frac{7}{8}}$	$\underline{\frac{11}{15}}$	$\underline{\frac{6}{13}}$	$\underline{62\frac{1}{2}}$	$\underline{6\frac{2}{5}}$

k. $91\frac{1}{8}$	l. $72\frac{3}{5}$	m. 75	n. $57\frac{1}{9}$	o. $18\frac{3}{4}$
$\underline{\frac{9}{10}}$	$\underline{24\frac{7}{9}}$	$\underline{18\frac{3}{4}}$	$\underline{\frac{4}{6}}$	$\underline{12\frac{1}{2}}$

p. $8\frac{6}{17}$	q. 21	r. 30	s. $144\frac{4}{19}$	t. $37\frac{1}{2}$
$\underline{3\frac{6}{7}}$	$\underline{41\frac{3}{4}}$	$\underline{10\frac{3}{7}}$	$\underline{98\frac{7}{19}}$	$\underline{6\frac{1}{4}}$

REVIEW

10. What is the area of a parallelogram whose base is 15.23 ft. long, and whose altitude is 19.64 ft.?

11. Multiply 1373.78 by 29, 36, 58, and 47.

12. Write in words 10056932812647829032.

13. What is the area of a rectangle 16 mi. by 32 mi.?

14. Change to decimals:

a. $\frac{2}{7}$	b. $\frac{3}{15}$	c. $\frac{9}{12}$	d. $\frac{1}{99}$	e. $\frac{49}{500}$
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15. In a town the tax rate for schools was .92 mills per dollar. What was the amount of the tax on a total valuation of \$9,875,632.58?

16. The distance of the sun from the earth is 93,000,000 miles. Find its ratio to the moon's distance, 240,000 miles.

ADDITION OF MIXED NUMBERS

1. Add
- $2\frac{1}{2}$
- ,
- $3\frac{3}{8}$
- ,
- $4\frac{5}{8}$
- ,
- $8\frac{3}{4}$
- ,
- $10\frac{2}{3}$
- ,
- $7\frac{5}{8}$
- .

$2 + 3 + 4 + 8 + 10 + 7 = 34$, the sum of the integers.

$$\begin{array}{r|rrrrr} 2 & 2 & 3 & 6 & 4 & 8 \\ 2 & 1 & 3 & 3 & 2 & 4 \\ 3 & 1 & 3 & 3 & 1 & 2 \\ \hline & 1 & 1 & 1 & 1 & 2 \end{array}$$

$2 \times 2 \times 3 \times 2 = 24$, the least common multiple of the denominators of the fractions.

$$\frac{1}{2} = \frac{12}{24} \quad \frac{3}{8} = \frac{16}{24} \quad \frac{5}{8} = \frac{20}{24} \quad \frac{3}{4} = \frac{18}{24} \quad \frac{2}{3} = \frac{16}{24} \quad \frac{5}{8} = \frac{15}{24}$$

$$\frac{12}{24} + \frac{16}{24} + \frac{20}{24} + \frac{18}{24} + \frac{16}{24} + \frac{15}{24} = \frac{97}{24} = 4\frac{1}{24}$$

$$34 + 4\frac{1}{24} = 38\frac{1}{24}$$

Another way is to reduce all the mixed numbers to improper fractions with a common denominator.

$$2\frac{1}{2} = \frac{5}{2} \quad 3\frac{3}{8} = \frac{11}{8} \quad 4\frac{5}{8} = \frac{29}{8} \quad 8\frac{3}{4} = \frac{35}{4} \quad 10\frac{2}{3} = \frac{32}{3} \quad 7\frac{5}{8} = \frac{61}{8}$$

$$\frac{5}{2} = \frac{60}{24} \quad \frac{11}{8} = \frac{33}{24} \quad \frac{29}{8} = \frac{116}{24} \quad \frac{35}{4} = \frac{210}{24} \quad \frac{32}{3} = \frac{256}{24} \quad \frac{61}{8} = \frac{183}{24}$$

$$\frac{60}{24} + \frac{33}{24} + \frac{116}{24} + \frac{210}{24} + \frac{256}{24} + \frac{183}{24} = \frac{913}{24} = 38\frac{1}{24}$$

2. Add
- $1\frac{1}{4}$
- ,
- $2\frac{5}{8}$
- ,
- $4\frac{1}{3}$
- ,
- $5\frac{3}{8}$
- ,
- $6\frac{2}{3}$
- ,
- $7\frac{1}{2}$
- .

3. Add
- $1\frac{1}{8}$
- ,
- $2\frac{3}{4}$
- ,
- $3\frac{3}{8}$
- ,
- $4\frac{1}{2}$
- ,
- $5\frac{1}{6}$
- ,
- $8\frac{1}{4}$
- .

4. Add
- $\$3\frac{3}{4}$
- ,
- $\$5\frac{1}{6}$
- ,
- $\$8\frac{3}{8}$
- ,
- $\$9\frac{1}{8}$
- ,
- $\$10\frac{1}{2}$
- .

5. A boy was sent by his father to collect three bills. His father gave him $\$2\frac{3}{4}$ in change; and he collected from one man $\$2\frac{1}{2}$, from another $\$6\frac{5}{10}$, and from another $\$10\frac{3}{8}$. How much money in all did he then have to hand to his father?

6. Prove the answer in 5 by reducing the money to decimal amounts, and adding.

FRACTION PROBLEMS

1. I bought cloth at $\$1\frac{7}{8}$ a yard, and sold it at $\$2\frac{1}{4}$ a yard. What did I gain on a yard?

2. From a barrel of vinegar containing $31\frac{1}{2}$ gallons, $12\frac{5}{8}$ gallons were drawn. How many gallons were left?

3. I purchased $10\frac{3}{4}$ tons of coal at one time, and $24\frac{5}{8}$ tons at another; after using $16\frac{7}{12}$ tons, how much remained?

4. A boy having $\$3\frac{1}{4}$, gave $\$1\frac{1}{5}$ for oranges, and $\$1\frac{1}{4}$ for a flag. How much money had he left?

5. A man spent $\$15\frac{7}{16}$ for provisions, and $\$12\frac{7}{8}$ for clothing. How much money had he left from $\$50\frac{1}{2}$?

6. A merchant bought two pieces of velvet, one containing $54\frac{9}{10}$ yards, and the other $56\frac{1}{12}$ yards. He sold $43\frac{3}{16}$ yards at one time, and $34\frac{1}{8}$ yards at another. How many yards had he left?

7. A grocer bought two tubs of butter, one tub containing $27\frac{1}{8}$ pounds, the other $32\frac{1}{6}$ pounds. After he sold $40\frac{1}{18}$ pounds, how many pounds had he left?

8. It is $95\frac{3}{4}$ feet from one house-lot on a city street to the opposite, and the sidewalks on the street are $12\frac{1}{2}$ feet wide. How wide is the street between the sidewalks?

9. The Capitol at Washington is 324 feet wide and $752\frac{1}{8}$ feet long. What is the difference between its length and its width?

10. From a piece of cloth containing $34\frac{3}{8}$ yards, there were sold $12\frac{3}{8}$ yards. How many yards were left?

11. There was a total eclipse of the sun on May 28th, 1900. The path of the eclipse crossed Alabama. The region in temporary darkness was $53\frac{7}{8}$ miles wide. If the eastern boundary of Alabama is $331\frac{1}{2}$ miles long, how many miles of the boundary were not in the path of the eclipse?

MULTIPLICATION OF FRACTIONS

Multiplying the numerator has what effect on the fraction? Dividing the denominator has what effect? Why?

If the numerator of the fraction $\frac{1}{8}$ be divided by 5, how will the fraction be affected?

If the numerator be multiplied by 4, how will the fraction be affected?

What difference will be made in the value of the fraction if the denominator is multiplied by 2? If the denominator is divided by 2?

1. Multiply $\frac{3}{17}$ by 4.

This means repeat $\frac{3}{17}$ four times.

$$\frac{3}{17} + \frac{3}{17} + \frac{3}{17} + \frac{3}{17} = \frac{12}{17}. \quad \frac{3}{17} \times 4 = \frac{12}{17}.$$

2. Multiply $\frac{11}{36}$ by 18.

$$\frac{11}{36} \times 18 = \frac{11}{\frac{36}{2}} \times \frac{1}{18} = \frac{11}{2} = 5\frac{1}{2}$$

First we cancel the factors common to denominator and multiplier. Then we multiply. The resulting fraction we reduce to a mixed number.

3. Multiply $\frac{7}{9} \times 6$.

$$\frac{7}{\frac{9}{3}} \times \frac{2}{6} = \frac{14}{3} = 4\frac{2}{3}$$

4. Multiply $1\frac{3}{4} \times 16$.

$$1\frac{3}{4} = \frac{7}{4} \quad \frac{7}{\frac{4}{1}} \times \frac{4}{16} = \frac{28}{1} = 28$$

5. Multiply:

a. $\frac{13}{24} \times 8$.	b. $\frac{5}{8} \times 4$.	c. $\frac{3}{16} \times 16$.	d. $\frac{47}{9} \times 7$.
e. $\frac{175}{45} \times 9$.	f. $\frac{127}{15} \times 5$.	g. $\frac{369}{147} \times 49$.	h. $\frac{21}{30} \times 15$.
i. $\frac{17}{40} \times 8$.	j. $\frac{161}{180} \times 45$.	k. $12\frac{2}{3} \times 8$.	l. $\frac{23}{7} \times 14$.

6. Multiply 24 by $\frac{3}{8}$.

$$\begin{array}{r} 3 \\ 24 \times \frac{3}{8} = 9 \\ 1 \end{array}$$

7. Multiply: a. 360 by $\frac{2}{3}$.b. 672 by $\frac{5}{6}$.c. 580 by $\frac{3}{4}$.d. 24 by $\frac{7}{8}$.

8. Find the products:

- a. $136 \times \frac{1}{3}$. b. $672 \times \frac{5}{6}$. c. $360 \times \frac{2}{3}$. d. $45 \times \frac{7}{9}$.
 e. $432 \times \frac{1}{4}$. f. $710 \times \frac{7}{8}$. g. $580 \times \frac{3}{4}$. h. $32 \times 2\frac{3}{8}$.
 i. $635 \times \frac{1}{5}$. j. $660 \times \frac{3}{26}$. k. $765 \times \frac{11}{12}$. l. $50 \times \frac{11}{14}$.
 m. $840 \times \frac{4}{8}$. n. $24 \times \frac{7}{8}$. o. $975 \times \frac{145}{8}$. p. $28 \times 3\frac{2}{3}$.
 q. $42 \times \frac{11}{12}$. r. $64 \times \frac{5}{8}$. s. $105 \times \frac{5}{7}$. t. $25 \times \frac{3}{4}$.

$$9. \frac{3}{4} \times 8 = \frac{3}{4} \text{ of } 8 = 8 \times \frac{3}{4}.$$

To multiply a whole number by a fraction is the same as to multiply a fraction by a whole number.

We may take three fourths of eight things or eight three-fourths of something, with the same result.

10. Discuss in this way:

- a. $9\text{¢} \times \frac{2}{3}$ or $\frac{2}{3}\text{¢} \times 9$. b. $27 \text{ bu.} \times \frac{1}{9}$ or $\frac{1}{9} \text{ bu.} \times 27$.
 c. $16 \text{ lb.} \times \frac{1}{8}$ or $\frac{1}{8} \text{ lb.} \times 16$. Would it make any difference in the total amount of tea used, if we put $\frac{1}{8}$ of 16 lb. in one package or $\frac{1}{8}$ lb. in each of 16 packages?
 d. $10 \text{ yd.} \times \frac{2}{5}$ or $\frac{2}{5} \text{ yd.} \times 10$. e. $60 \text{ min.} \times \frac{1}{4}$ or $\frac{1}{4} \text{ min.} \times 60$.

11. Multiply 15 minutes by $\frac{4}{5}$.

This means, take four fifths of fifteen minutes.

$$15 \div 5 = 3.$$

One fifth of fifteen minutes is three minutes.

$3 \text{ min.} \times 4 = 12 \text{ min.}$ Three minutes multiplied by four is twelve minutes.

12. Discuss multiplying:

- a. 200 qt. by $\frac{3}{10}$. b. 30 days by $\frac{5}{6}$. c. 365 days by $\frac{1}{5}$.

FRACTIONS OF FRACTIONS

1. Multiply
- $\frac{3}{5}$
- by
- $\frac{2}{7}$
- .

This means that the seventh part of $\frac{3}{5}$ is to be taken twice. The seventh part of $\frac{3}{5}$ is $\frac{3}{35}$. Twice $\frac{3}{35}$ is $\frac{6}{35}$.

$$\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$$

The product of the two fractions is equivalent to the product of the numerators divided by the product of the denominators.

$$3 \times 2 = 6$$

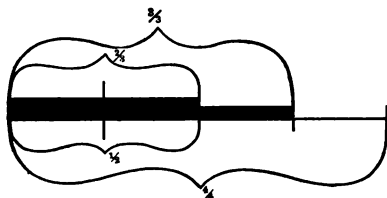
$$5 \times 7 = 35$$

$\frac{3}{5} \times \frac{2}{7}$ may be written as $\frac{3}{5}$ of $\frac{2}{7}$, or $\frac{2}{7}$ of $\frac{3}{5}$.

2. Multiply
- $\frac{2}{3}$
- by
- $\frac{3}{4}$
- .

$$\frac{2}{3} \times \frac{3}{4} = \frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$$

This means, take two thirds of three fourths.



In canceling the common factors, it is best to write 1 when it is a quotient. This helps us not to overlook any of the numbers or factors.

WRITE

3. Multiply
- $\frac{3}{5}$
- by
- $\frac{4}{7}$
- by
- $\frac{6}{11}$
- .

This means, take $\frac{6}{11}$ of $\frac{4}{7}$ of $\frac{3}{5}$, or $\frac{3}{5} \times \frac{4}{7} \times \frac{6}{11} = ?$

$$\frac{3}{5} \times \frac{4}{7} \times \frac{6}{11} = \frac{3 \times 4 \times 6}{5 \times 7 \times 11} = \frac{72}{385}$$

4. Multiply:

$$\begin{array}{llll} a. \frac{4}{5} \times \frac{10}{16}. & b. \frac{3}{5} \times \frac{5}{7}. & c. \frac{4}{9} \times \frac{18}{27}. & d. \frac{8}{11} \times \frac{55}{66}. \\ e. \frac{7}{9} \times \frac{63}{77}. & f. \frac{13}{14} \times \frac{28}{120}. & g. \frac{11}{17} \times \frac{68}{120}. & h. \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}. \\ i. \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}. & j. \frac{3}{7} \times \frac{7}{9} \times \frac{8}{15}. & k. \frac{5}{6} \times \frac{4}{7} \times \frac{6}{11}. & l. \frac{3}{4} \times \frac{4}{5} \times \frac{7}{10}. \\ m. \frac{5}{12} \times \frac{9}{20} \times \frac{6}{27}. & n. \frac{7}{11} \times \frac{22}{23} \times \frac{46}{9}. & o. \frac{14}{27} \times \frac{9}{28} \times \frac{6}{13} \times \frac{26}{30}. \end{array}$$

5. Multiply together $\frac{4}{9}$, $\frac{3}{11}$, $6\frac{2}{7}$, $9\frac{3}{5}$, $2\frac{1}{2}$, 63. These numbers may be written:

$$\frac{4}{9} \times \frac{3}{11} \times \frac{44}{7} \times \frac{48}{5} \times \frac{5}{2} \times \frac{63}{1} = \frac{4}{\underset{1}{9}} \times \frac{3}{\underset{1}{11}} \times \frac{44}{\underset{1}{7}} \times \frac{48}{\underset{1}{5}} \times \frac{5}{\underset{1}{2}} \times \frac{63}{\underset{1}{1}}$$

$$= \frac{2 \times 3 \times 4 \times 1 \times 48}{1 \times 1 \times 1 \times 1 \times 1} = 1152$$

6. Find the value of:

a. $\frac{1}{2} \times \frac{3}{7} \times \frac{2}{11} \times \frac{4}{81} \times 9$. b. $3 \times 7\frac{1}{3} \times 1\frac{1}{5} \times 3\frac{8}{11}$.

7. What will $12\frac{3}{4}$ pounds of spice cost at $\$ \frac{8}{9}$ a pound?

8. What will $3\frac{1}{2}$ bushels of onions cost at $\$ \frac{7}{8}$ a bushel?

9. At 75¢ a bushel, what will $1\frac{1}{5}$ bu. of corn cost?

10. If a man earns $\$ 1\frac{7}{8}$ in one day, how much can he earn in 24 days?

11. $\frac{1}{179} \times 18\frac{7}{11} \times 9\frac{3}{5} \times \frac{1}{2} \times \frac{3}{4} \times 7 \times \frac{3}{6} \times \frac{11}{4} \times 25 = ?$

12. At \$16 per M, what will $\frac{1}{2}$ of a thousand feet of lumber cost?

13. If 1 yd. of cloth cost $\$ 1\frac{1}{4}$, what will $6\frac{1}{3}$ yd. cost?

14. What will $3\frac{3}{8}$ bu. of peaches cost at $\$ 2\frac{1}{2}$ per bu.?

15. If a quantity of lumber is valued at $\$ 75\frac{8}{15}$, what will be the value of $\frac{5}{12}$ of it?

16. If A pays \$56 as railroad fare for a certain distance, how much should he pay for $\frac{3}{4}$ of that distance?

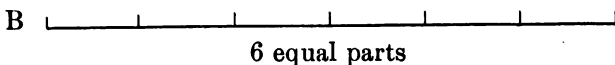
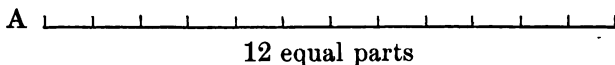
17. If a roll of wire is worth $\$ \frac{5}{6}$, what is $\frac{2}{3}$ of a roll worth?

18. I owned $\frac{7}{15}$ of a vessel and sold $\frac{9}{20}$ of my share. What part of the vessel did I sell?

19. A tailor used $\frac{9}{10}$ of a yard of silesia and $\frac{5}{12}$ as much lasting. How much lasting did he use?

20. $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{6}{25}$ of \$50 = ?

DIVISION OF FRACTIONS



In A each part is $\frac{1}{12}$ the size of each part of B.

In B each part is 2 times the size of each part of A.

$$\frac{1}{12} \text{ is } \frac{1}{2} \text{ of } \frac{1}{6} \qquad \frac{1}{6} = 2 \times \frac{1}{12} \qquad \frac{1}{12} = \frac{1}{2} \times \frac{1}{6}$$

We may divide a fraction by a whole number by multiplying the denominator by that number.

If $\frac{1}{6} \div 2 = \frac{1}{12}$, then $\frac{2}{6} \div 2 = \frac{2}{12}$.

We may divide a fraction by a whole number by dividing the numerator by that number.

$$\frac{3}{12} \div 3 = \frac{3}{36} \text{ or } \frac{1}{12}. \quad \text{But } \frac{3}{36} = \frac{1}{12}.$$

Dividing the numerator of a fraction by a number or multiplying the denominator by it divides the fraction.

1. Divide:

- a. $\frac{1}{8}$ by 3. b. $\frac{3}{25}$ by 5. c. $\frac{8}{21}$ by 4. d. $\frac{15}{100}$ by 5.
e. $\frac{7}{8}$ by 4. f. $\frac{9}{10}$ by 3. g. $\frac{18}{20}$ by 10. h. $\frac{75}{100}$ by 25.

2. Six boys had to run a relay race of two miles. What part of a mile did each boy run?

3. Five boys had a quarter of a dollar. What part of a dollar did each boy have as his share?

4. Three boys had a half dollar. They spent each his share. What part of a dollar did each spend?

5. How much is one half of a half dollar? one fifth?

MULTIPLICATION BY MIXED NUMBERS

1. What is the product of
- $3\frac{2}{3} \times 4\frac{7}{8} \times 1\frac{5}{2}$
- ?

$$3\frac{2}{3} = \frac{11}{3} \quad 4\frac{7}{8} = \frac{39}{8} \quad 1\frac{5}{2} = \frac{7}{2}$$

$$\frac{11}{3} \times \frac{39}{8} \times \frac{7}{2} = \frac{13 \times 39}{8 \times 2} = \frac{13 \times 15}{8 \times 2} = \frac{195}{16} = 12\frac{3}{16}$$

We reduce the mixed numbers to improper fractions, cancel the factors found in both the numerators and the denominators, multiply together the remaining values of the numerators for the numerator of the product and of the denominators for the denominator of the product, and reduce the result to lowest terms.

2. Find the products:

- | | |
|--|---|
| a. $5\frac{6}{7} \times 3\frac{1}{2}$. | q. $11\frac{2}{7} \times 8\frac{4}{5}$. |
| b. $67 \times 9\frac{1}{2}$. | r. $108 \times 12\frac{4}{5}$. |
| c. $34\frac{1}{3} \times \frac{1}{5} \times 68$. | s. $30\frac{5}{6} \times 44\frac{4}{20}$. |
| d. $\frac{1}{2} \times 75 \times \frac{2}{3} \times 28$. | t. $\frac{6}{7} \times 2\frac{1}{2} \times \frac{3}{5} \times 61$. |
| e. $6\frac{1}{5} \times 2\frac{7}{9} \times \frac{1}{4} \times 12$. | u. $\frac{2}{3} \times 16\frac{1}{8} \times \frac{6}{7} \times 9\frac{1}{3}$. |
| f. $12\frac{3}{4} \times 11\frac{5}{6}$. | v. $2\frac{3}{7} \times 65\frac{1}{3} \times 46\frac{1}{6}$. |
| g. $8\frac{5}{11} \times 6\frac{2}{7}$. | w. $\frac{7}{8} \times \frac{9}{10} \times 20 \times 25\frac{1}{2}$. |
| h. $12\frac{2}{3} \times 9$. | x. $\frac{7}{9} \times 3\frac{4}{81} \times 4\frac{1}{2} \times \frac{2}{5} \times 6\frac{1}{7} \times 5$. |
| i. $4\frac{7}{8} \times 9\frac{1}{4}$. | y. $\frac{7}{8} \times \frac{5}{6} \times \frac{7}{16}$. |
| j. $15\frac{3}{4} \times 20\frac{8}{9}$. | z. $14 \times 15\frac{1}{16} \times 3\frac{5}{6}$. |
| k. $1475 \times \frac{7}{9} \times \frac{21}{100}$. | aa. $\frac{9}{10} \times 8\frac{3}{4} \times \frac{9}{11} \times 1\frac{1}{12}$. |
| l. $\frac{800}{12} \times \frac{3}{8} \times \frac{1000}{9}$. | bb. $\frac{6}{7} \times \frac{11}{8} \times \frac{6}{38} \times 21 \times \frac{3}{5}$. |
| m. $13\frac{3}{8} \times 17\frac{7}{9}$. | cc. $\frac{1}{4} \times 8 \times \frac{2}{7} \times 19$. |
| n. $7\frac{1}{8} \times 8\frac{3}{4}$. | dd. $\frac{9}{10} \times 7 \times \frac{11}{15} \times 87\frac{3}{11}$. |
| o. $2\frac{1}{3} \times \frac{2}{3} \times \frac{3}{4} \times 85$. | ee. $3\frac{2}{3} \times 4\frac{7}{8} \times 15$. |
| p. $6\frac{3}{4} \times 2\frac{1}{2}$. | ff. $\$7 \times \frac{3}{5} \times 2\frac{1}{4} \times \frac{1}{10} \times 8$. |

DIVISION OF FRACTIONS

1. How many times is $\frac{1}{3}$ contained in $\frac{3}{4}$? or, $\frac{3}{4} \div \frac{1}{3} = ?$

If we were to divide $\frac{3}{4}$ by 1, our answer would be $\frac{3}{4}$. But our divisor is only $\frac{1}{3}$ of 1. Since $1 = 3 \times \frac{1}{3}$, our quotient is $3 \times \frac{3}{4} \div 1$.

$$\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4}$$

2. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$?

$$\frac{3}{4} \div \frac{1}{3} = \frac{9}{4} \quad \frac{2}{3} = 2 \times \frac{1}{3} \quad \frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$$

3. Divide $\frac{5}{7}$ by $\frac{3}{8}$.

$$\frac{5}{7} \div 3 = \frac{5}{7 \times 3} = \frac{5}{21} \quad \frac{5}{21} \div \frac{1}{8} = \frac{5}{21} \times \frac{8}{1} = \frac{40}{21}$$

$$\frac{5}{7} \div \frac{3}{8} = \frac{5 \times 8}{7 \times 3} = \frac{40}{21} = 1\frac{19}{21}$$

When we divide a fraction by a fraction, we multiply the numerator of the dividend by the denominator of the divisor for the numerator of the quotient, and the denominator of the dividend by the numerator of the divisor for the denominator of the quotient.

To divide a fraction by a fraction invert the divisor and multiply.

4. Divide $\frac{7}{12}$ by $\frac{4}{9}$.

$$\frac{7}{12} \div \frac{4}{9} = \frac{7 \times 9}{12 \times 4} = \frac{\overset{21}{\cancel{63}}}{\underset{16}{\cancel{48}}} = \frac{21}{16} = 1\frac{5}{16}$$

5. Divide:

- | | | | |
|--------------------------------------|--------------------------------------|--|---------------------------------------|
| a. $\frac{4}{7}$ by $\frac{9}{10}$. | e. $\frac{1}{7}$ by $\frac{8}{9}$. | i. $\frac{8}{15}$ by $\frac{12}{20}$. | m. $\frac{2}{3}$ by $\frac{7}{8}$. |
| b. $\frac{1}{6}$ by $\frac{1}{6}$. | f. $\frac{3}{4}$ by $\frac{1}{3}$. | j. $\frac{7}{9}$ by $\frac{6}{7}$. | n. $\frac{5}{9}$ by $\frac{1}{6}$. |
| c. $\frac{7}{8}$ by $\frac{9}{10}$. | g. $\frac{2}{3}$ by $\frac{27}{8}$. | k. $\frac{9}{18}$ by $\frac{11}{16}$. | o. $\frac{8}{11}$ by $\frac{7}{22}$. |
| d. $\frac{5}{6}$ by $\frac{6}{7}$. | h. $\frac{1}{9}$ by $\frac{3}{10}$. | l. $\frac{9}{20}$ by $\frac{12}{18}$. | p. $\frac{9}{10}$ by $\frac{2}{7}$. |

DIVISION OF FRACTIONS

In multiplying a fraction by a fraction we multiply the numerators together and the denominators together.

We divide a fraction by a fraction by inverting the terms of the divisor and proceeding as in multiplication of fractions.

1. Divide $\frac{5}{8}$ by $\frac{2}{9}$.

$\frac{5}{8} \div \frac{2}{9}$ means

1st. In $\frac{5}{8}$ are how many ninths? 9 times as many eighths as one unit contains ninths; that is, $\frac{5}{8} \times \frac{9}{1} = \frac{45}{8}$.

2d. In $\frac{45}{8}$ are how many combinations of 3-ninths? Evidently $\frac{1}{3}$ as many as there are single or 1-ninths; that is, $\frac{45}{8} \div 3 = \frac{15}{8} = 1\frac{7}{8}$.

2. What do these expressions equal?

a. $\frac{6}{7} \div \frac{5}{8}$. b. $\frac{2}{3} \div \frac{5}{6}$. c. $\frac{1}{9} \div \frac{7}{25}$. d. $\frac{3}{8} \div \frac{4}{9}$.

3. Divide:

a. $\$ \frac{3}{4}$ by $\frac{1}{8}$. b. $\$ \frac{3}{8}$ by $\frac{1}{20}$. c. $\frac{1}{10}$ gal. by $\frac{3}{4}$. d. $\frac{1}{25}$ T. by $\frac{3}{5}$.

4. If $\frac{1}{6}$ yard of velvet cost $\$ \frac{3}{4}$, how much will 1 yard cost?

$$\$ \frac{3}{4} \div \frac{1}{6} = \$ \frac{3}{4} \times \frac{6}{1} = \$ \frac{9}{2} = \$ 4\frac{1}{2} = \$ 4.50.$$

5. Discuss each step in 4.

DIVISION OF MIXED NUMBERS

In order to divide mixed numbers we first reduce the mixed numbers to improper fractions.

6. Divide $2\frac{1}{2}$ by $1\frac{3}{4}$.

$$2\frac{1}{2} \div 1\frac{3}{4} = \frac{5}{2} \div \frac{7}{4} = \frac{5}{2} \times \frac{4}{7} = \frac{10}{7} = 1\frac{3}{7}$$

7. Divide:

a. $8\frac{1}{4}$ by $6\frac{3}{5}$. b. $3\frac{1}{3}$ by $\frac{1}{2}\frac{0}{1}$. c. $9\frac{2}{5}$ by $1\frac{3}{8}$. d. $7\frac{1}{2}$ by $3\frac{1}{3}$.
e. $4\frac{2}{3}$ by $1\frac{1}{7}$. f. $1\frac{9}{10}$ by $3\frac{1}{5}$. g. $1\frac{1}{6}$ by $5\frac{1}{5}$. h. $2\frac{1}{2}$ by $1\frac{1}{4}$.

REVIEW OF FRACTIONS

1. Divide:

- a.* 6 by $1\frac{1}{4}$. *f.* $\frac{1}{3}\frac{5}{8}$ by 20. *k.* 212 by $1\frac{5}{16}$. *p.* $19\frac{1}{2}$ by $1\frac{1}{8}$.
b. $\frac{1}{18}$ by 6. *g.* $\frac{405}{19}$ by 15. *l.* 63 by $1\frac{8}{7}$. *q.* $73\frac{1}{2}$ by $9\frac{1}{8}$.
c. $\frac{1}{11}$ by 6. *h.* $\frac{1}{6}\frac{2}{5}\frac{1}{1}$ by 27. *m.* 420 by $\frac{9}{11}$. *r.* $54\frac{1}{8}$ by $25\frac{1}{8}$.
d. $\frac{42}{91}$ by 12. *i.* $\frac{45}{5}\frac{9}{1}$ by 75. *n.* $1\frac{3}{4}$ by 5. *s.* $15\frac{1}{4}$ by $30\frac{1}{2}$.
e. $\frac{1}{3}\frac{8}{7}$ by 9. *j.* 14 by $\frac{7}{8}$. *o.* $8\frac{1}{8}$ by $\frac{3}{8}$. *t.* $\frac{3}{1000}$ by $1\frac{1}{2}$.

2. If 1 yard of percale cost $\$ \frac{3}{8}$, how many yards can be bought for $\$ \frac{9}{16}$?

3. What number multiplied by $5\frac{1}{3}$ will give 146 for the product?

4. The dividend is $520\frac{1}{5}$ and the quotient is $36\frac{2}{10}$. What is the divisor?

5. What number is that which multiplied by $\frac{5}{8}$ of $\frac{2}{7}$ of $15\frac{1}{8}$ will produce $\frac{5}{8}$?

6. A man spent $\$165\frac{3}{10}$ in $8\frac{1}{2}$ weeks. How much did he spend in 1 week?

7. At $\$9\frac{3}{8}$ a cord, how many cords of wood can be bought for $\$138\frac{3}{4}$?

8. I spent $\$3\frac{1}{4}$ in 8 days. How much did I spend in 1 day at that rate?

9. If 8 yards of serge cost $\$7\frac{1}{2}\frac{1}{8}$, what is the price a yard?

10. If $\frac{1}{6}$ of a tub of butter sells for $\$10\frac{3}{8}$, what should a tub sell for?

11. What will 493 lb. of tea cost at $\$ \frac{5}{8}$ per lb.?

12. What will 1286 bu. of apples cost at $\$ \frac{2}{3}\frac{1}{8}$ per bu.?

13. How much money will 227 yd. of cloth cost at $\$ \frac{3}{4}$ per yd.?

14. What will be the cost of 8750 sq. ft. of land at $\$ \frac{7}{10}$ per sq. ft.?

PRECEDENCE OF SIGNS

Parentheses (), brackets [], braces { }, and the vinculum —, when used to inclose numbers, are called **symbols of aggregation**. Two or more numbers inclosed by these symbols or covered by the vinculum are treated as single numbers. Operations indicated by signs between inclosed numbers must be performed before operations on numbers outside the symbols.

In any expression in which the signs $\times$, $\div$, $+$, $-$, occur, division and multiplication are performed before addition and subtraction, unless a symbol of aggregation indicates a different order of operations.

$$63 \div 7 + 2 = 9 + 2 = 11;$$

but

$$63 \div (7 + 2) = 63 \div 9 = 7.$$

$$\frac{63}{7} + 2 = 9 + 2 = 11, \text{ and } \frac{63}{7 + 2} = \frac{63}{9} = 7.$$

When no sign occurs between the first parenthesis and the first number inclosed, the plus sign is regarded as standing before the number. In the equation $16 - (3 + 4) = 9$, the number 3 is considered as having the plus sign before it.

1. Compare $16 - (3 + 4) = 9$ with $16 - 3 + 4 = 17$.
2. Explain :
 - a. $5 \times 7 + 8 + 3 \times 2 = x$. b. $16 \div 4 + 8 \times 3 - 4 - 10 = x$.
3. Express by symbols and figures:
 - a. From 34 take the sum of 25 and 3.
 - b. To 30 add the difference between 25 and 8.
 - c. To the difference between 30 and 25 add 3.
4. To 10 add 6 and from the sum take 7 divided by 4.

ROMAN NOTATION

1. Read: I, V, X, L, C, D, M.

LXXIII	CXLIV	DXI
CLXIII	DCLXIV	CCCI
MDCCCLXXII	DCCCXLIII	XVIII
XLIX	MMDCCCLVIII	CCCXX
CCXCVI	LX	XLVII
MCD	XXXIII	LXXIV
MMMXXX	XLIV	DCCLXXVII
XCII	XCIX	MI
CXCIX	CDLXXVIII	MDXXXIX
LXVIII	CMX	CCCXXXIII

2. A bar, called a *vinculum*, over a letter increases its value a thousand times. $\overline{V} = 5000$.

3. Read:

$\overline{X}$	$\overline{XC}$	$\overline{VM}$
$\overline{XXVMMXXVII}$	$\overline{XLCDXLIV}$	$\overline{MDCCCXCIX}$
$\overline{MDCCXCI}$	$\overline{MCMXXXIII}$	$\overline{VCMLXXVII}$
$\overline{MMCCXXII}$	$\overline{VMMMDCCCLXXXVIII}$	

4. Express in Roman notation:

202, 47, 91, 80, 20, 77, 101, 10, 111, 606, 437, 908, 899, 763, 497, 829, 827, 999, 888, 850, 2333, 3232, 3333, 4321, 1234, 5678, 8765, 9999, 25,671, 891,347, 912,342, 1,678,942, 16,713, 191,919, 29,134, 23,476, 912,345, 1,678,942, 3,456,713, 9,308,275.

5. The next to the last page in the preface of a book was numbered xxxiv. What should be the Arabic numerals upon the last page of the preface?

6. What Roman numerals are placed upon the corner-stones of buildings this year?

REVIEW

1. A man sold .375 of his farm. How much of it had he left?
2. How many sevenths of a pint are there in 3 pints?
3. If 35 marbles be divided equally among 5 boys, how many marbles will 2 of the boys get?
4. A man who had \$1000 in the bank withdrew \$450.25. How much had he there then?
5. A grocer who had a hogshead of molasses lost 215 thousandths of it by leakage. How much was left?
6. When a circle is divided into 5 equal parts, what is one part called? What are 4 of the parts called?

MULTIPLICATION DRILL TABLE

In the table below give the product of each pair of consecutive digits, both in columns and in rows.

With such a row as 4, 2, 6, 9, 5, 0, 1, 8, 6, 4, 3, 5, say only "8, 12, 54, 45, 0, 0, 8, 48, 24, 12, 15."

	A	B	C	D	E	F	G	H	I	J	K	L
I	2	4	6	8	9	7	6	3	1	4	9	7
II	7	1	5	4	1	8	2	6	4	7	2	1
III	3	7	6	1	7	1	5	8	9	0	1	2
IV	1	8	3	4	5	6	2	7	8	9	3	4
V	9	4	1	1	8	1	9	3	4	5	6	7
VI	4	3	9	8	7	2	0	2	3	4	5	6
VII	5	6	2	8	8	2	1	8	9	0	2	3
VIII	6	2	1	8	9	2	2	5	6	7	8	9
IX	8	1	8	9	0	2	3	4	2	1	8	4
X	7	0	1	9	1	2	6	7	1	8	2	4
XI	1	5	5	2	9	3	6	7	8	9	0	3
XII	4	8	7	6	3	3	3	4	5	6	7	8

QUESTIONS

1. If 3 bbl. of apples cost \$4, what will 1 bbl. cost?
2. If 3 yards of cloth cost \$2, what part of a dollar will 1 yard cost? How much will 2 yards cost?
3. How many barrels of vinegar, at \$7 a barrel, can one buy for \$15? for \$16? for \$20?
4. How many yards of silk, at \$6 a yard, can one buy for \$7? for \$9? for \$10? for \$14?
5. What number is $\frac{4}{8}$ of 8? What is $\frac{5}{8}$ of 8? $\frac{7}{8}$ of 8?
6. If a melon cost 16¢, what is $\frac{1}{8}$ of the melon worth? What are $\frac{2}{8}$ of it worth? $\frac{5}{8}$? $\frac{7}{8}$?
7. How many quarts of milk, at 6¢ a quart, can one buy for 16¢? for 17¢? for 18¢?
8. If gingham is 9¢ a yard, how many yards can one buy for 10¢? for 17¢? for 18¢? for 20¢?
9. If apples cost 10¢ a quart, what is $\frac{1}{10}$ of a quart worth? $\frac{3}{10}$? $\frac{5}{10}$? $\frac{7}{10}$?
10. At \$10 a yard, how much velvet can one buy for \$11? for \$14? for \$19?
11. If it requires 70 hours to perform a piece of work, in how many days can it be done by working 7 hours a day? by working 8 hours a day? by working 9 hours a day? by working 10 hours a day?
12. How many eighths of a cent are there in 24¢?
13. How many ninths of an orange are there in 32 oranges?
14. If a yard of cloth cost \$3, how many yards can one buy for \$4? How many for \$5?
15. What is the area of a rectangle whose base is $3\frac{1}{2}$ miles and whose altitude is 4 miles?

Draw this on the blackboard: scale 3 in. = 1 mi.

ADDITION

1. The United States military forces were distributed as follows in 1899 (Dec. 10):

In the United States	34,299 men,
In Porto Rico	3,363 men,
In Cuba	11,187 men,
In Philippine Islands	32,315 men,
En route to Philippine Islands	17,099 men,
In Alaska	499 men,
In Hawaiian Islands	466 men.

How many soldiers were there in all ?

2. In 1899 there arrived in the United States from :

Austria-Hungary	62,491 immigrants,
Germany	17,476 immigrants,
Italy	77,419 immigrants,
Russia	60,982 immigrants,
Sweden	12,797 immigrants,
United Kingdom	45,181 immigrants.

What was the total number of immigrants from these countries ?

3. In the year 1899 the United States government expended the following sums of money:

War Dept.	\$ 229,841,254.
Navy Dept.	63,942,104.
Pensions	139,384,929.
Miscellaneous	119,191,255.
Ordinary Expenses	565,175,254.
Interest	39,896,925.

What was the total amount expended ?

ADDITION PROBLEMS

1. How many men were there in an army composed of 75,600 infantry, 25,087 cavalry, 14,097 artillery, 6095 men in the engineer corps, 1095 men in the medical corps, and 1278 in the commissary department?

2. How many bricks are there in seven different heaps of 7069, 25,804, 716, 4684, 97,605, 120,176, and nine thousand seventy bricks each?

3. A wholesale grocery firm had in stock seven thousand ninety-six pounds of rice, twenty thousand four pounds of sugar, 918 pounds of pepper, 4079 pounds of cocoa. What was the total weight of all these goods?

4. An army had ten thousand men in the ranks, 2019 sick, 1409 home on furlough, 4060 had been killed and wounded, 619 were prisoners. How many men were in the army at first?

5. There were brought into the port of Boston, in one day, eight hundred thousand nine herring, 7016 bluefish, 508 haddock, 20,016 cod, 1219 shad, 619 hake, and nineteen hundred mackerel. How many fish in all were brought in?

6. A coal district yielded 10,609 tons on Monday, 7018 tons on Tuesday, 20,109 tons on Wednesday, 9600 tons on Thursday, 14,700 tons on Friday, 16,849 tons on Saturday. What was the weekly output of coal?

7. In one year, San Francisco sent 49,684 barrels of flour to Asiatic ports, 29,344 barrels to South American ports, 10,968 barrels to islands in the Pacific, and 390,897 barrels to other parts of the globe. How many barrels in all were sent away?

8. Add: \$10.07, 90¢, \$8.92, \$10, \$5.01.

DIVISION OF DECIMALS

1. Find the quotients to three places of decimals:

- | | |
|----------------------------------|-----------------------------------|
| <i>a.</i> $.817 \div .9147$. | <i>i.</i> $61.123 \div .0146$. |
| <i>b.</i> $.213 \div 91.614$. | <i>j.</i> $9.1534 \div .000716$. |
| <i>c.</i> $9378.92 \div 9.789$. | <i>k.</i> $.0467 \div .01471$. |
| <i>d.</i> $.793 \div .49$. | <i>l.</i> $918 \div 914.71$. |
| <i>e.</i> $8.8 \div .0641$. | <i>m.</i> $43 \div .0006947$. |
| <i>f.</i> $7147.12 \div 1127$. | <i>n.</i> $967.104 \div 12.046$. |
| <i>g.</i> $78.1 \div 1.071$. | <i>o.</i> $91.671 \div .000916$. |
| <i>h.</i> $91.142 \div 7.8$. | <i>p.</i> $3.26 \div .111$. |

2. Divide:

- a.* 3.012 by .0006. *b.* 293916.669 by 541.283.
c. 130.4 by .0004. *d.* by 4. *e.* 46.634205 by 4807.65.
f. 1.69 by 1.3. *g.* by .13; *h.* by 13; *i.* and by .013.
j. .00281 by 1.405. *k.* by 1405. *l.* and by .001405.
m. 72.36 by 36; *n.* by .0036. *o.* .003 by 1.6.
p. 6725402.3544 by 7089. *q.* and by .7089.

3. A grocer paid \$9.45 for a barrel of brown sugar, weighing 315 lb. What was the cost per pound?

4. At the price of \$5.75 a ton for coal, how many tons does a bill for \$155.25 represent?

5. At the price of \$.59125 per bushel, how many bushels does a sale amounting to \$4730 represent?

6. At the price of \$22.25 per ton, what number of tons does a sale amounting to \$77.875 represent?

7. $10.5 \times .05 \div 2.5 = ?$ 8. $25.16 \div .08 \times .3 = ?$

9. $$.384 \times 7.5 \div .5 = ?$ 10. $$.125 \times .10 \times 6 = ?$

11. Draw to the scale of $\frac{1}{8}$ in. to 1 ft. a parallelogram, whose area is 100 sq. ft. and whose altitude is 16.6 ft.

REVIEW

Add together :

1. 4.13295, 42.171, .3721, .0064, .6, 25.
2. 3010.3, 21.747, .006887, .96, 325.37, 141.2.
3. .0065, 65, .065, 6.5, .000065, .65.
4. 16.5, 176.1105, 11.0012, 18.75, 11.5, 14.123.
5. .813, 72.5, 32.071, 2.1574, 371.4, 2.75.
6. 7.1, 3275, 3.2675, 1.00005, 14.7, .001.

Find the prime factors :

7.	8.	9.	10.	11.
84	90	100	112	117
1760	1690	2520	2432	121
75	98	147	495	132
135	168	196	125	144
385	624	945	231	1575
1296	1331	1617	1188	1050

12. Multiply each number in column A by each number in B.

A	B
Multiplicands	Multipliers
a. 95,080	f. 870
b. 40,009	g. 706
c. 76,900	h. 918
d. 81,118	i. 118
e. 48,484	j. 896

Prove each answer by treating each product as a dividend and each multiplier as a divisor.

PROBLEMS

1. If $\frac{2}{3}$ of a vessel is worth \$880, what is the value of $\frac{3}{11}$ of the vessel?

2. A man has $\frac{2}{7}$ of a farm of 4000 acres left to him. He sells $\frac{2}{3}$ of his share. How many acres has he left?

3. If 16 horses eat 80 bushels of corn in 36 days, in how many days will 6 horses eat 45 bushels?

16 horses eat 80 bushels in 36 days.

Then 16 horses would eat 1 bushel in $(36 \div 80)$ days.

Also 1 horse would eat 1 bushel in $\frac{36 \times 16}{80}$ days.

And 6 horses would eat 45 bushels in $\frac{6 \times 1 \times 9}{36 \times 16 \times 45}$ days,
 $= 6 \times 1 \times 9 = 54$ days.

4. If 25 horses eat 100 bu. of corn in 30 days, in how many days will 500 horses eat 1000 bu.?

There are several different ways of finding the answers to 3 and 4.

5. If 858 men in 6 months consume 234 barrels of flour, how many barrels will be consumed by 979 men in three months?

6. How many persons should share $1\frac{1}{2}$ lb. of candy so that each person has $\frac{1}{8}$ lb.? $\frac{1}{4}$ lb.? $\frac{1}{6}$ lb.?

7. What will $\frac{2}{5}$ T. of sugar cost at \$4 $\frac{3}{8}$ per cwt.?

8. I bought sugar, and sold it at \$ $\frac{1}{4}$ per bbl. advance, gaining \$180. How many barrels did I buy?

9. A has 88 $\frac{1}{2}$ ¢, B has 18 $\frac{3}{4}$ ¢. What part of B's money is A's money?

10. A corporation having its capital divided into 6000 shares, voted to represent its property by 20,000 shares. What part of an old share was a new one?

REVIEW

1. In how many days can 54 men do the work that 1 man can do in 1198 days?

2. If a man's income is \$5110 a year, how much is it a day? a week? a month?

3. If a steamer goes in 6 days from an Atlantic port to a port in Western Europe, 3070 miles, how many miles does the steamer go daily?

4. A man bought 156 pounds of prunes for \$10.92, and sold them for 9¢ a pound. How much did he gain?

5. The product of two numbers is 51,679,780, and one of the factors is 615. What is the other factor?

6. A dealer has 5100 pounds of butter, and wishes to pack it in 60 tubs. How much must he put in each tub?

7. X bought a number of horses at \$74 each, paying \$17,484 in all for them. How many horses did he buy?

8. Find the value of:

a. $1\frac{3}{4} - 1\frac{1}{9}$. b. $2\frac{1}{4} - \frac{7}{15}$. c. $7\frac{1}{3} - 6\frac{7}{8}$. d. $84\frac{7}{15} - 16\frac{1}{3}$.
e. $4\frac{1}{7} - \frac{3}{9}$. f. $9\frac{1}{2} - 3\frac{7}{8}$. g. $246\frac{3}{8} - 164\frac{1}{5}$. h. $16\frac{5}{8} - 5\frac{3}{8}$.

9. Since 63 gallons make a hogshead, how many hogshead do 2898 gallons make?

10. A California orchard of 700 A. yielded in one year the following quantities of fruit: apricots, 339,411 lb.; peaches, 2,115,314 lb.; nectarines, 210,518 lb.; pears, 280,124 lb.; plums, 4705 lb.; prunes, 22,283 lb. What was the average number of pounds of fruit per A.?

11. A sold 4 T. of hay @ \$12 per T., 80 bu. of wheat @ \$1 per bu., and took in payment a cow worth \$65, a wagon worth \$40, and cash. How much did he receive?

12. Draw a triangle with base 7 in. and altitude $4\frac{1}{2}$ in. Find its area.

SCHOOL PER CENTS

1. John had 87% in his arithmetic, 60% in reading, 80% in manual work, 75% in drawing, 70% in music, 60% in spelling, and 90% in Nature study. What was his average, if each study counted the same?

$$\begin{array}{r}
 87\% \\
 60 \\
 80 \\
 75 \\
 70 \\
 60 \\
 \hline
 90 \\
 522
 \end{array}$$

$$7 \overline{)522} \\ 74\frac{4}{7}\%$$

2. But what was his per cent if arithmetic counted 3 points, reading and manual work 2 points each, and the other exercises 1 point each?

$$87\% \times 3 = 261$$

$$60 \times 2 = 120$$

$$80 \times 2 = 160$$

$$75 \times 1 = 75$$

$$70 \times 1 = 70$$

$$60 \times 1 = 60$$

$$90 \times 1 = 90$$

$$\begin{array}{r} 11 \quad 836 \end{array}$$

$$\begin{array}{r}
 11 \overline{)836} \\
 76\%
 \end{array}$$

3. Find the averages of various reports.

4. In a class of 36 boys and girls $33\frac{1}{3}\%$ were girls. How many girls were there? How many boys? 50% were not absent in October. How many came every day? 25% were tardy once each during the entire school year. How many were tardy?

5. In another class of 45 boys and girls $33\frac{1}{3}\%$ were girls. How many boys were there? How many girls? 80% were not absent in October. How many came every day? 9 were tardy once during the year. What per cent of 45 was that? What is the ratio of 9 to 45? $\frac{1}{5}$ is what per cent of 100?

REVIEW

1. A dealer sold 4 suits for \$25.50 a suit, 12 vests for \$2.12½ each, and 3 coats for \$7.25 each. How much did he get for all the garments sold?

2. What are 25 webs of cloth, each web containing 872 yards, worth, at 6½¢ a yard?

3. X bought, at 37½¢ per lb., 9 sacks of copper, each sack containing 72 lb. What was the total cost of the copper?

4. Subtract :

a. 30604	b. 31024	c. 40679	d. 70686	e. 89706	f. 69807
19087	10013	30416	30425	46302	14306
<u>19087</u>	<u>10013</u>	<u>30416</u>	<u>30425</u>	<u>46302</u>	<u>14306</u>
g. 14071	h. 10102	i. 19006	j. 70601	k. 11001	l. 30608
12618	8016	8709	5908	10101	7909
<u>12618</u>	<u>8016</u>	<u>8709</u>	<u>5908</u>	<u>10101</u>	<u>7909</u>

5. A man willed to his wife \$25,000, to his two sons \$19,375 each, to his three daughters \$7750 each, and to a hospital \$3460. What was the sum of the legacies?

6. Add :

7. Copy, point in periods, and read :

a. 792085579	b. 92837465321	c. 186457900
909672916	9739765689079	920749365
1251495987650	31859678009678	283467458729
868253926	9412287064571493	814382257
92150470279	45902768537	8369376914936
3234918065933	45794555672990	21743790051
2672409308	63749568	847090439
5368542984945	64573594693845	50210706090
64709365197	5277456789987654	739634721987
3447615994883772	49596878397	68598765432
185938976	7395084309724606	119647896
934172706050896	29684371989	5846952081638
685675	23678521	79749327965
<u>891076813</u>	<u>139324680357</u>	<u>574981672</u>

RATIOS OF NUMBERS

1. What part of 4 is 1?

1 is one fourth of 4, because 4 times 1 is 4.

The part one number is of another is its ratio to that number.

2. What part:

a. of 6 is 1? b. of 9 is 1? c. of 16 is 1? d. of 245 is 1?

3. What part of 8 is 3?

Since $1 = \frac{1}{8}$ of 8, 3 must equal $\frac{3}{8}$ of 8.

4. What part:

a. of 9 is 4? b. of 3 is 2? c. of 6 is 5? d. of 11 is 7?

e. of 8 is 5? f. of 13 is 4? g. of 18 is 15? h. of 9 is 6?

5. What part of 7 is 9?

Since $1 = \frac{1}{7}$ of 7, 9 must equal $\frac{9}{7}$ of 7, or $1\frac{2}{7}$ of 7.

6. What part:

a. of 9 is 13? b. of 47 is 63? c. of 10 is 87?

d. of 39 is 19? e. of 19 is 39? f. of 413 is 527?

7. What part:

a. of 9 ft. is 1 ft. ? b. of 4 cwt. is 3 cwt. ?

c. of 4 m. is 3 m. ? d. of 10 da. is 7 da. ?

e. of 4 lb. is 3 lb. ? f. of 8 yr. is 5 yr. ?

8. What part of $\frac{4}{7}$ is $\frac{3}{7}$?

$\frac{3}{7}$ is the same part of $\frac{4}{7}$ that 3 is of 4, which is $\frac{3}{4}$.

9. What part:

a. of $\frac{14}{163}$ is $\frac{13}{163}$? b. of $\frac{7}{13}$ is $\frac{4}{13}$? c. of $\frac{4}{177}$ is $\frac{3}{177}$?

d. of $\frac{5}{11}$ is $\frac{3}{11}$? e. of $\frac{14}{19}$ is $\frac{13}{19}$? f. of $\frac{23}{58}$ is $\frac{13}{58}$?

10. What part of 1 bushel is 3 pecks?

1 bu. = 4 pk. 3 pk. = $\frac{3}{4}$ of 4 pk. 3 pk. = $\frac{3}{4}$ of 1 bu.

11. What part:

a. of 1 yd. is 2 ft. ? b. of 1 ft. is 1 in. ? c. of 1 T. is 13 cwt. ?

FRACTIONAL PARTS OF NUMBERS

1. What is $\frac{7}{8}$ of 5459?

First Method. $\frac{7}{8}$ of 5459 = 7 times $\frac{1}{8}$ of 5459.

$\frac{1}{8}$ of 5459 is $682\frac{3}{8}$, and 7 times $682\frac{3}{8} = 4776\frac{5}{8}$.

Second Method. $\frac{7}{8}$ of 5459 = $\frac{1}{8}$ of 7 times 5459.

7 times 5459 = 38,213, and $\frac{1}{8}$ of 38,213 = $4776\frac{5}{8}$.

$$\begin{array}{r} \text{First} \\ \text{Method} \end{array} \quad \begin{array}{r} 8 \overline{) 5459} \\ \underline{682\frac{3}{8}} \\ 7 \\ \underline{4776\frac{5}{8}} \end{array}$$

$$\begin{array}{r} \text{Second} \\ \text{Method} \end{array} \quad \begin{array}{r} 5459 \\ 7 \\ 8 \overline{) 38213} \\ \underline{4776\frac{5}{8}} \end{array}$$

2. What is the value of :

a. $\frac{6}{7}$ of 3843?

b. $\frac{3}{5}$ of 7148?

c. $\frac{6}{7}$ of 46,935?

d. $\frac{5}{8}$ of 7987?

e. $\frac{1}{12}$ of 58,437?

f. $\frac{1}{144}$ of 87,516?

FINDING NUMBERS FROM FRACTIONAL PARTS

3. $5861 = \frac{1}{7}$ of what number?

If $5861 = \frac{1}{7}$ of some number, then $\frac{7}{7}$ must equal 7 times 5861, which is 41,027.

Proof. $\frac{1}{7}$ of 41,027 = 5861.

4. Of what number :

a. does $3498 = \frac{1}{8}$? b. does $\frac{3}{4}\frac{1}{8} = \frac{1}{9}$? c. does $59,387 = \frac{1}{5}$?

d. does $58.46 = .1$? e. does $\frac{1}{4}\frac{3}{9}\frac{7}{8} = \frac{1}{6}$? f. does $327.93 = .0001$?

THE EQUATION

An **equation** is an expression of equality between two quantities.

$$7x = 14. \quad 15(12 - 5 - 4) = x + 2(18 \div 9).$$

5. If $8321 = \frac{1}{5}$ of a number, what is the number?

$$\text{Let } x = \text{the number} \quad \frac{x}{5} = 8321 \quad x = 8321 \times 5 = 41,605$$

6. If $3497 = \frac{1}{3}$ of a number, what is the number?

FINDING NUMBERS FROM PARTS

- 1.
- $3476 = \frac{8}{9}$
- of what number?

Let x = the number.

$$\frac{8}{9} \text{ of } x = 3476 \quad \frac{8x}{9} = 3476 \quad 8x = 9 \times 3476$$

$$x = \frac{9 \times 3476}{8} = 9 \times 434\frac{4}{8} = 9 \times 434\frac{1}{2} = 3910\frac{1}{2}$$

2. Of what number does :

$$a. 5496 = \frac{6}{7} ? \quad b. 8372 = \frac{7}{10} ? \quad c. 2475 = \frac{3}{10} ?$$

CHANGES IN THE TERMS OF A FRACTION

3. Reduce
- $\frac{3}{4}$
- to an equivalent fraction, having 6 for a numerator.

The required numerator, 6, is 2 times the given numerator 3.

We multiply both terms by the same number.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

4. Reduce
- $\frac{5}{7}$
- to an equivalent fraction, having 10 for its numerator.

5. Reduce
- $\frac{6}{12}$
- to an equivalent fraction, having 9 for its numerator.

6. Reduce
- $\frac{9}{10}$
- to an equivalent fraction, having 6 for its numerator.

7. Reduce
- $\frac{3}{7}$
- to twenty-firsts.

$$21 = 3 \times 7$$

$$\frac{3}{7} \times \frac{3}{3} = \frac{9}{21}$$

8. Reduce :

- a. $\frac{3}{8}$ to twenty-fourths. b. $\frac{4}{5}$ to fiftieths. c. $\frac{2}{3}$ to ninths.
 d. $\frac{4}{5}$ to thirtieths. e. $\frac{7}{9}$ to twenty-sevenths.
 f. $\frac{8}{15}$ to forty-fifths. g. $\frac{4}{13}$ to sixty-fifths.
 h. $\frac{6}{7}$ to forty-ninths. i. $\frac{7}{11}$ to twenty-seconds.

QUESTIONS

1. How many vest patterns, each containing $\frac{3}{4}$ yd., can be cut from a piece of vesting containing $25\frac{1}{4}$ yd.?

2. How many baskets, each holding $\frac{7}{8}$ of a peck, can be filled from $82\frac{3}{8}$ pecks of corn?

3. How many books, at $\$ \frac{5}{6}$ each, can be bought for $\$232\frac{1}{6}$?

4. How many baskets, each holding $\frac{1}{3}\frac{5}{2}$ of a bushel, can be filled from $453\frac{3}{2}$ bushels of apples?

5. How many barrels, holding $2\frac{1}{4}$ bushels each, can be filled from $59\frac{3}{4}$ bushels of apples?

6. What part of a ton of coal may be purchased for $\$5\frac{1}{16}$, at the rate of $\$6\frac{3}{4}$ a ton?

7. How long would it take a bicyclist to ride $125\frac{5}{7}$ miles, if he rode $11\frac{6}{14}$ miles an hour?

8. How many pieces of cloth, each piece $1\frac{3}{4}$ yards long, can be cut from a bolt of cloth containing $31\frac{1}{2}$ yards?

9. How long will it take 11 men to build a wall that 1 man can build in $15\frac{8}{9}$ days?

10. If $\frac{4}{5}$ of a barrel of flour cost $4\frac{7}{8}$, what is the price per barrel?

11. How many doz. eggs at $23\frac{1}{2}\text{¢}$ per doz. will pay for 10 lb. of butter at $21\frac{1}{4}\text{¢}$ per lb.?

12. If $\frac{3}{4}$ of $\frac{5}{6}$ of a farm is worth $\$2540$, what is the value of the whole farm?

13. If $\frac{5}{6}$ of the value of a bicycle is $\$36\frac{1}{3}$, what is the bicycle worth?

14. If $\frac{7}{8}$ of $\frac{3}{4}$ of a pound of butter will last a family 1 day, how long will $9\frac{5}{14}$ pounds last them?

15. If 7 companies in a regiment muster 630 men, what will be the muster of 10 companies of similar size?

QUESTIONS

1. If $1\frac{1}{2}$ T. of hay cost \$22 $\frac{7}{8}$, how much will $9\frac{3}{5}$ T. cost?
2. I bought a bolt of silk for \$96, and sold it for $\frac{5}{8}$ of the cost, thereby losing \$2 per yard. How many yards were there in the bolt?
3. From a bolt of muslin containing $37\frac{3}{4}$ yd., there were cut at different times $9\frac{7}{8}$ yd., $6\frac{1}{2}$ yd., $3\frac{5}{8}$ yd., and $7\frac{2}{3}$ yd. How many yards of muslin were left?
4. What number subtracted from $14\frac{3}{8}$ will leave $1\frac{3}{4}$ for a remainder?
5. What will $7\frac{1}{2}$ pounds of fish cost at $5\frac{3}{4}$ ¢ a pound?
6. What will $6\frac{7}{8}$ yards of braid cost, at $12\frac{1}{2}$ ¢ a yard?
7. What will $7\frac{2}{3}$ yd. of flannelet cost, at $9\frac{3}{4}$ ¢ per yd.?
8. What will $12\frac{1}{2}$ pounds of tea cost, at $62\frac{1}{2}$ ¢ a pound?
9. What will 15 pounds of beef cost, at $9\frac{1}{2}$ ¢ a pound?
10. At \$ $\frac{7}{8}$ a yard, what will $2\frac{3}{5}$ yards of cheviot cost?
11. At $3\frac{1}{5}$ ¢ a yard, how much will 15 rolls of braid cost, each roll containing 7 yards?
12. Find the value of $14\frac{4}{5}$ barrels of molasses, at \$ $10\frac{2}{3}$ a barrel.
13. We may solve such questions as these in dollars and cents as well as in parts of dollars. In 13, page 89, \$ $\frac{2}{3}$ = 60¢. $60\text{¢} \times 7\frac{3}{4} = ?$ Or we may reduce the answer \$ $41\frac{3}{10}$ to \$ and ¢. \$ $41\frac{3}{10}$ = \$4.65.
14. If the sum of two fractions is $\frac{5}{8}$, and one of the fractions is $\frac{2}{9}$, what is the other fraction?
15. A, B, and C buy a vessel for \$6300; A pays $\frac{2}{7}$ of the cost, B, $\frac{4}{7}$, and C the remainder. What did each one pay?

16. How many yards of carpet, $\frac{3}{4}$ of a yard wide, will cover a hall $20\frac{1}{2}$ yards long and $1\frac{1}{2}$ yards wide?

Draw to scale on paper or blackboard a picture to illustrate this.

17. What is the divisor if the dividend is $\frac{7}{9}$ and the quotient $\frac{4}{9}$?

18. If a man earns \$ $1\frac{1}{5}$ a day, how many days must he work to earn \$ $36\frac{2}{3}$?

19. A person owning $\frac{3}{7}$ of property worth \$2000, sold $\frac{2}{3}$ of his share. How many dollars' worth did he retain?

20. A man having \$240, paid debts as follows: To A, $\frac{1}{3}$ of the money; B, $\frac{1}{10}$; C, $\frac{1}{8}$; and D, $\frac{1}{6}$. How many dollars did he pay A? B? C? D? How many dollars did he have left?

21. A speculator invested \$3740 in copper-mining shares, and found at the end of 3 years that he had gained \$ $156\frac{1}{3}$ more than $33\frac{1}{3}\%$ of his investment. How much did he gain yearly?

22. A and B engaged in a speculation. A furnished $\frac{7}{12}$ of the capital, and B the rest; if B had furnished \$492 $\frac{2}{3}$ more, their shares would have been equal. What sum did each invest?

23. In one day a merchant made \$63, which was $\frac{7}{8}$ of what he made the next day. $\frac{5}{8}$ of what he made in the second day was just 4 times what he made in the third day. How much did he make in the three days?

24. A cattle dealer had $\frac{1}{5}$ of his cattle in one pen, $\frac{1}{4}$ in another pen. The rest of the cattle, 77 in number, were in a third pen. How many cattle had he in the three pens?

25. What are $7\frac{3}{4}$ bu. of peaches worth at $\frac{1}{5}$ of \$14 per bu.?

26. At \$ $\frac{1}{4}$ a yard, how many yards of gingham can be bought for \$5 $\frac{7}{8}$?

BILLS

A **bill of goods** is a written statement of articles sold, including the price of each article, the names of debtor and of creditor, and the amount of the indebtedness.

The **debtor** is the one who owes the debt, and the **creditor** is the one to whom the debt is owed.

A *bill* is receipted by writing the words "*Received payment*" or "*Paid*" at the bottom of the bill or across the face, with the signature of the *creditor* written beneath. If a clerk or other authorized agent writes the signature, he adds the word "Per" or "By," and his own initials.

1. Find the total amount of this bill :

ROCHESTER, N.Y., Aug. 1, 1900.

W. H. VICKS,

Bought of SAMUEL MARCY,

DEALER IN FINE GROCERIES.

2 lb. Tea,	.80		
3 " Coffee,	.30		
10 " Sugar,	.06		
1 " Pepper,	.40		
1 " Chocolate,	.80		
10 bars Soap,	.05		

Received Payment,

SAMUEL MARCY,

Per C. H. M.

2. Make out bills for groceries.
3. Make out bills for meats and provisions.
4. Make out bills for dry goods.

1.

PHILADELPHIA, Oct. 2, 1900.

MRS. GEORGE CHASE,

Bought of JAMES MARSHALL, DEALER IN DRY GOODS.

8 yd. of Velvet,	\$5.00		
10 " " Muslin,	.20		
12 " " Golf Plaid,	1.25		
3 " " Percale,	.40		
20 " " Dimity,	.50		

2.

DETROIT, June 14, 1900.

THOMAS HALLECK,

Bought of WILLIAM HOLMES,
MEATS AND PROVISIONS.

6 lb. Sirloin,	.25		
10 " Roast Beef,	.15		
8½ " " Lamb,	.12		
15¼ " " Turkey,	.25		
4 " Suet,	.08		

Paid June 15, W. HOLMES, *By* R. H. S.

3.

BANGOR, ME., June 29, 1900.

CHARLES FESSENDEN,

Bought of ISRAEL POTTER,

6500 ft. Hemlock Boards,	\$6.00 per M.		
1050 " Sills,	5.00 " "		
8000 " Joists,	6.00 " "		
7200 " Pine Boards,	11.00 " "		

REVIEW

1. George Mitchell bought of Vaile & Tripp, New York, Apr. 14th, 1900, 3 lb. Bl. Tea, 65¢; 13 lb. Prunes, 11¢; Apr. 16th, 5 lb. Lard, 9¢; Apr. 18th, 3 bars Soap, 23¢; 3 boxes Starch, 15¢; Apr. 21, 1 Bath-brick, 8¢; 3 doz. Eggs, 17¢; 4 lb. Butter, 19¢; Apr. 23d, 12 lb. Flour, 3¢; 1 box Soda Biscuits, 30¢; Apr. 25th, 4 lb. Currants, 8¢, and 7 lb. Raisins, 9¢. On Apr. 21st the sum of \$5 was paid on the above account, and the balance was paid on May 1st. Make out a receipted bill.

	<i>Dr.</i>	<i>Cr.</i>	
2. Received \$2157, \$2543,	\$2157	\$312	} Subtrahends
and \$1626.	2543	265	
Paid \$312, \$265, \$763.	1626	763	
What is the balance in hand?	Balance	4986	Remainder
	\$6326	\$6326	Minuend

3. Received \$3162, \$2154, \$3172, and paid \$2163, \$84, \$13; find the balance in hand.

4. Received \$7000, and paid \$213, \$715, \$610, \$216; find the balance in hand.

If there are several amounts to be added, and several to be subtracted, the amounts may first be classified, and those of like kind added together. Then the difference of their sums may be found.

5. Received \$25, paid \$19, received \$700, paid \$16, \$200, and \$31, and received \$15; find the balance.

6. At the beginning of a year a railroad had 19,615 cars. During the year 13 cars were smashed in accidents, 115 broken up, and 210 sold. 560 new cars were made. How many cars had the railroad at the end of the year?

7. A train going 45 miles an hour passed another going 36 miles an hour. How far apart would the trains be in two hours, if they were going in opposite directions?

REVIEW

1. $2\frac{5}{8}$ of $\frac{4}{5}$ of 5 = ?

3. $\frac{5}{9} \times 3\frac{3}{4} \times 1\frac{4}{5} = ?$

2. $\frac{1}{3}$ of $\frac{2}{5}$ of $2\frac{1}{2} = ?$

4. $\frac{5}{9}$ of $\frac{3}{11}$ of 4 = ?

5. A man gave away \$10,561. He gave $\frac{2}{7}$ of it to a library, $\frac{1}{3}$ of the remainder to his brother, and the rest he divided equally among his 7 children. What sum did each of his children receive?

6. Find the value of:

a. $6 \div \frac{1}{2}$. *d.* $36 \div \frac{6}{11}$. *g.* $12 \div \frac{3}{5}$. *j.* $12 \div \frac{9}{5}$.
b. $12 \div \frac{4}{5}$. *e.* $6 \div \frac{2}{3}$. *h.* $42 \div 2\frac{2}{3}$. *k.* $8 \div \frac{2}{3}$.
c. $12 \div \frac{2}{3}$. *f.* $12 \div \frac{6}{5}$. *i.* $12 \div \frac{1}{5}$. *l.* $63 \div 1\frac{5}{2}$.

7. Divide:

a. .9375 by .075. *d.* 18 by .288. *g.* 15.875 by .0025.
b. 215.7 by 86.28. *e.* 25.2835 by 7.81. *h.* 7.3926 by 2.22.
c. 24.75 by .25. *f.* 11.83 by .845. *i.* 568.944 by 3.24.

8. Change: *a.* 5 to 8ths. *b.* 13 to 4ths. *c.* 8 to 32ds.
d. 9 to 8ths. *e.* 5 to 5ths. *f.* 6 to 4ths.

9. Change: *a.* $\frac{5}{8}$ to 36ths. *b.* $\frac{2}{7}$ to 49ths. *c.* $\frac{6}{17}$ to 51sts.

10. There were 7178 A. of allspice or pimento berry plantations in Jamaica in 1898. The total yield of berries was 6,857,858 lb. What was the average yield per A.?

11. A Western farmer cut three crops of alfalfa from his meadows. The yield was 7 tons to the acre and was sold at \$14.00 a ton. How much did he get for one season's crop on 12 acres?

12. Southern California, during the year 1899, exported to the Atlantic States 12,000 car loads of oranges. If each car contained 334 boxes, and each box held 75 pounds of fruit, how many pounds of oranges were exported?

ARITHMETIC

Arithmetic is essential to *business* and to *science*.

Arithmetic is the science and the art of measuring magnitudes and of counting numbers.

A **magnitude** is any thing.

A *line* has one dimension of magnitude, **length**.

A *surface* has two dimensions, **length** and **breadth**.

A *solid* has three dimensions, **length**, **breadth**, and **thickness**. *Thickness* may be height or depth.

A **number** counts things or units.

Magnitude tells, **How much?**

Number tells, **How many?**

Like numbers have units of the same magnitude or kind : three desks and five desks ; seven ounces and two ounces ; four boys and nine boys.

Unlike numbers have units of different magnitudes or kinds : ten feet and eight dollars ; fifteen pounds and eleven hours.

A number is **concrete** when it is applied to one or more magnitudes or objects : forty dollars ; twenty-three yards.

A number is called **abstract** when it is not applied to any magnitude or object : six, twelve, two hundred.

The relative value of the magnitude to the unit is its **ratio**. All ratios are abstract numbers.

The ratio of the magnitude ten acres to the unit one acre is the abstract number ten. The ratio of one dollar to the unit three dollars is one third. The ratio of seventeen dozen eggs to the unit one dozen is seventeen.

Notation is a method of writing numbers.

Numeration is a method of reading numbers.

We use commonly two kinds of notation, the Arabic and the Roman, but only one kind of numeration, the decimal, which is the basis of both kinds of notation.

TERMS AND PROCESSES

Addition finds the sum of two or more numbers.

The numbers to be added together are called **addends**.

The sign, +, *plus*, placed between two numbers means that the numbers are to be added together.

$9 + 6$, *nine plus six*, means, Add nine and six together.

1. Read: $7 + 5 + 10$; $9 + 900 + 9,000,000$; $M + C + V$.

The sign, =, read *equals* or *equal*, is called the sign of **equality**.

$7 + 8 = 15$, is read *seven plus eight equals fifteen*.

An expression of equality is termed an **equation**. The expression, $7 + 8 = 15$, is an equation.

x may represent *anything*, or What? $10 + 12 = x$.
 $5 + 6 + 8 = x$.

Subtraction finds the difference between two numbers. Only **like numbers** can be added or subtracted.

The number from which another number is taken is the **minuend**.

The number taken from the *minuend* is the **subtrahend**.

The number resulting from *subtraction* is the **difference** or *remainder*.

The sign, -, *minus*, placed between two numbers means that the first number is the minuend and the second is the subtrahend.

$9 - 5$, *nine minus five*, means, From nine take five.

2. Read: $10 - 6$; $18 - 13$; $11,005 - 9985$; $D - CC$.

3. Read and answer:

a. $C + XL + IX - VII = ?$

b. $250 + 150 + 75 + 125 + 400 - 1000 = ?$

c. $500 + D - M + 70 + V = ?$

d. $85 + 25 + 40 - 20 = 50 + x$.

TERMS AND PROCESSES

Multiplication repeats one number as many times as there are units in another number.

The **multiplicand** is the number to be repeated.

The **multiplier** is the number which tells how many times the *multiplicand* is to be repeated.

The **product** is the number resulting from the *multiplication*.

The **factors** of a number are the numbers that, multiplied together, give the number.

The *multiplicand* and *multiplier* are *factors* of the *product*.

The sign of multiplication, $\times$, is read, *multiplied by*.

15×3 is read, *Fifteen multiplied by three*.

Division finds how many times one number is contained in another.

The number divided by another number is the **dividend**.

The number dividing another number is the **divisor**.

The number denoting *how many times* the dividend contains the divisor is the **quotient**.

The sign $\div$ is read, *divided by*.

$20 \div 5$ is read, *Twenty divided by five*.

The *divisor* and *quotient* correspond with the *factors* in *multiplication*; and the *dividend* corresponds with the *product*.

Division finds one of the two factors of a product when the other factor is known.

If the dividend does not contain the divisor exactly, we have either a *quotient* with a **remainder** or a *fractional quotient*.

To prevent confusion in processes it is better to call the result of subtraction the *difference* than to call it the *remainder*.

Multiplication is a short method of adding the same number to itself two or more times.

Division is a short method of subtracting the same number several or many times from another number.

Divisor $\times$ quotient = dividend.

Product $\div$ multiplier = multiplicand.

The *multiplicand* and *product* are like numbers.

The *multiplicand* may be *abstract* or *concrete*.

The *multiplier* is always an *abstract* number.

The *product* is always of the same kind as the *multiplicand*.

The *dividend* and *quotient* are like numbers.

The *divisor*, *dividend*, and *quotient* may be *abstract* or *concrete*.

The multiplicand is always supposed to be in substance a number of things, or a concrete number. This is also true of the dividend.

In division by *partition* the divisor is abstract, or a ratio; but in division by *measurement* the divisor is concrete.

1. Why cannot we divide 9 qt. by 2 pt.?
2. Why cannot we multiply 15 yd. by 20 yd.?
3. Why cannot we add 10 cu. ft. to 3 cu. yd.?
4. What must we do to answer the problems above?
5. Divide 98 cu. ft. by 8.

A. $8 \overline{)98}$ cu. ft.

12 cu. ft. + 2 cu. ft. *remainder*.

This is an inexact result.

B. $8 \overline{)98}$ cu. ft.

$12\frac{2}{8}$ cu. ft. = $12\frac{1}{4}$ cu. ft. This is an exact result with a fractional quotient.

In *B* the remainder, seen in *A*, is itself divided by 8, and appears in the fraction $\frac{2}{8}$.

PERCENTAGE

per cent means hundredth or hundredths.

7 per cent = 7 hundredths.

$22\frac{1}{2}$ per cent = $22\frac{1}{2}$ hundredths.

The sign, %, is employed in commercial transactions to represent the words "per cent."

EQUIVALENTS

WORDS		SYMBOLS		DECIMALS		COMMON FRACTIONS
1 per cent	=	1 %	=	.01	=	$\frac{1}{100}$
2 per cent	=	2 %	=	.02	=	$\frac{2}{100}$
3 per cent	=	3 %	=	.03	=	$\frac{3}{100}$
4 per cent	=	4 %	=	.04	=	$\frac{4}{100}$
5 per cent	=	5 %	=	.05	=	$\frac{5}{100}$
6 per cent	=	6 %	=	.06	=	$\frac{6}{100}$
7 per cent	=	7 %	=	.07	=	$\frac{7}{100}$
8 per cent	=	8 %	=	.08	=	$\frac{8}{100}$
9 per cent	=	9 %	=	.09	=	$\frac{9}{100}$
10 per cent	=	10 %	=	.10	=	$\frac{10}{100}$
20 per cent	=	20 %	=	.20	=	$\frac{20}{100}$
50 per cent	=	50 %	=	.50	=	$\frac{50}{100}$
100 per cent	=	100 %	=	1.00	=	$\frac{100}{100}$
125 per cent	=	125 %	=	1.25	=	$\frac{125}{100}$
2 %	=	$\frac{1}{50}$				
4 %	=	$\frac{1}{25}$				
5 %	=	$\frac{1}{20}$				
10 %	=	$\frac{1}{10}$				
$6\frac{1}{4}$ %	=	$\frac{1}{16}$				
$18\frac{3}{4}$ %	=	$\frac{3}{16}$				
$31\frac{1}{4}$ %	=	$\frac{5}{16}$				
$8\frac{1}{3}$ %	=	$\frac{1}{12}$				
$41\frac{2}{3}$ %	=	$\frac{5}{12}$				
$58\frac{1}{3}$ %	=	$\frac{7}{12}$				
$12\frac{1}{2}$ %	=	$\frac{1}{8}$				
$37\frac{1}{2}$ %	=	$\frac{3}{8}$				
$62\frac{1}{2}$ %	=	$\frac{5}{8}$				
$87\frac{1}{2}$ %	=	$\frac{7}{8}$				
$16\frac{2}{3}$ %	=	$\frac{1}{6}$				
$83\frac{1}{3}$ %	=	$\frac{5}{6}$				
20 %	=	$\frac{1}{5}$				
25 %	=	$\frac{1}{4}$				
75 %	=	$\frac{3}{4}$				
$33\frac{1}{3}$ %	=	$\frac{1}{3}$				
$66\frac{2}{3}$ %	=	$\frac{2}{3}$				
50 %	=	$\frac{1}{2}$				
$3\frac{1}{3}$ %	=	$\frac{1}{30}$				
$2\frac{1}{2}$ %	=	$\frac{1}{40}$				

1. What is 4 per cent of \$3? \$5? \$8? \$10? \$12?
2. What is 5 per cent of \$6? \$8? \$12? \$100?
3. What is 6 per cent of \$12? \$20? \$50? \$150?
4. What is 7 per cent of \$10? \$15? \$20? \$2500?
5. A farmer having a flock of 40 sheep lost 5 per cent of them. How many sheep had he left?

6. Express as decimal fractions :

WRITE

- | | | |
|------------------------------|------------------------------|-------------------------------|
| a. 9 per cent. | i. $9\frac{3}{8}$ per cent. | q. $7\frac{1}{2}$ per cent. |
| b. $4\frac{1}{2}$ per cent. | j. 20 per cent. | r. $10\frac{2}{5}$ per cent. |
| c. $29\frac{1}{4}$ per cent. | k. 46 per cent. | s. $9\frac{5}{8}$ per cent. |
| d. $8\frac{1}{5}$ per cent. | l. 12 per cent. | t. $103\frac{1}{2}$ per cent. |
| e. $9\frac{3}{4}$ per cent. | m. 175 per cent. | u. 225 per cent. |
| f. $\frac{3}{8}$ per cent. | n. $6\frac{1}{4}$ per cent. | v. $\frac{3}{4}$ per cent. |
| g. $2\frac{5}{8}$ per cent. | o. $8\frac{3}{4}$ per cent. | w. $\frac{2}{3}$ per cent. |
| h. $\frac{1}{5}$ per cent. | p. $33\frac{1}{3}$ per cent. | x. $\frac{4}{5}$ per cent. |

7. Express as common fractions in their lowest terms :

- | | | |
|------------------------------|------------------------------|------------------------------|
| a. 22 per cent. | c. $31\frac{1}{5}$ per cent. | e. $27\frac{1}{3}$ per cent. |
| b. $28\frac{3}{8}$ per cent. | d. $51\frac{1}{4}$ per cent. | f. 325 per cent. |

8. Write with the per cent sign, % :

- a. .07. b. .61. c. 1.47. d. .06. e. 1.06. f. 1.11.

When the decimal fraction is smaller than .01, it may be represented by the sign % with the number before it expressed either decimally or by a common fraction.

$$.056 = 5\frac{6}{10}\% = 5.6\% = \text{also } 5\frac{3}{5}\%.$$

$$.0095 = \frac{95}{100}\% = .95\% = \text{also } \frac{19}{20}\%.$$

9. Write with the sign, % :

- | | | | | |
|----------|------------|-----------|-----------|------------|
| a. .675. | c. 1.217. | e. .1355. | g. .2775. | i. .011. |
| b. .035. | d. .00125. | f. .0005. | h. 12.3. | j. .00575. |

10. Admission for adults was 50¢, but for children it was 70% as much. What was it?

PERCENTAGE

1. A landlord sold a house for 20% more than its cost, \$3000. What was the selling price?

<i>First</i>	$20\% = \frac{1}{5}$	<i>Second</i>	\$3000
<i>Method</i>	$\frac{1}{5}$ of \$3000 = \$600	<i>Method</i>	<u>.20</u>
	\$3000 + \$600 = \$3600	<i>Gain</i>	\$600.00

2. A dealer paid for goods \$1825, and sold them so that he gained 12%. How much did he gain?

3. A lawyer collected \$362.56, and charged 5% for his services. How much did he receive for his work?

4. A farmer having 1500 sheep, lost 25% of them. How many did he lose?

5. A man having \$1960 in a bank, drew out 20% of it. How much had he left in the bank?

6. A dealer imported 1500 boxes of oranges, and $12\frac{1}{2}\%$ of them decayed. How many boxes did he lose, and how many had he left?

7. I bought a stock of goods for \$4565, and paid $3\frac{1}{2}\%$ for freight. What was the whole cost of the goods?

8. A man's salary is \$2000 a year, and he saves $37\frac{1}{2}\%$ of it. How much does he spend?

9. A man inherited \$20,000 and spent 40% of it. How much had he left?

10. Two men engaged in business with \$18,250 capital apiece; one gained 20% and the other lost 20% the first year. What was the capital of each man then?

11. What is the difference between 6% of \$1040 and 7% of \$905?

12. What per cent of \$200 is \$80?

13. What per cent of \$1250 is \$250?

USES OF PERCENTAGE

The principles of percentage are very helpful in **insurance, taxes, borrowing money, commissions, customs, and duties.** These are very important subjects in *business* and *government*.

1. If the tax rate in the city is $1\frac{7}{10}\%$, what is the amount of tax to be paid on a house assessed at \$4000?

2. At the same rate what is the tax on a \$9000 house?

3. The duty on silks was 40% of their value. What was the amount to be paid on \$19,500 worth?

4. What is the annual premium on insurance for \$39,000, at $2\frac{1}{8}\%$?

5. A farmer insures his barns and their contents to the amount of \$17,800. What premium does he pay at $\frac{1}{2}\%$?

6. If the interest of \$1 for 1 yr. is 6¢, what will be the interest of \$5 for the same time? of \$8? of \$12? of \$15? of \$20? of \$30? of \$50? of \$100? of \$125?

7. If the interest of \$1 for 1 yr. is 6¢, what will be the interest of \$1 for 2 yr.? for 3 yr.? for 4 yr.? for 5 yr.?

8. A man took out a policy of life insurance for \$5000 at \$28 per \$1000 premium. What was the amount of the premium? What per cent was this?

9. A farmer sent \$1250 worth of produce to a merchant to sell at 5% commission. What was the amount of the commission, if all the produce was sold?

10. What is 12% of \$1358?

11. What is 50% of \$500,000?

12. What is .05% of \$1,200,000?

Find out from your father and friends what you can about taxes, insurance, etc.

SYMBOLS OF DIVISION

Division may be expressed by placing the divisor *under* the dividend with a short line between them. The expression $\frac{72}{9}$ shows that 72 is to be divided by 9. It is read "72 divided by 9," or, seventy-two ninths.

1. Read the following expressions:

a. $54 \div 9 + 15 = 5 \times 3 + 6$.

b. $29 + \frac{15}{5} = 40 \div 5 \times 4$.

c. $48 \div 6 = 63 - 15 + 12 \times 2$.

Division is also indicated by the sign $\div$; thus, $9)72$ is equivalent to $72 \div 9$, or to $\frac{72}{9}$.

RATIO AND DIVISION

The quotient, which is the result of division, tells the **ratio** of the dividend to the divisor.

Division is also expressed by the ratio sign.

$20:5$ *twenty to five*, is equivalent to $20 \div 5$; $\frac{20}{5}$; or $5)20$.

2. What is the ratio of 27 to 3? of 3 to 54?

3. What is the ratio of 7 to 11? of 11 to 14?

4. What is the ratio of 9 to 27? of 27 to 18?

5. What is the ratio of 42 to 5? of 5 to 84?

6. What is the ratio of 72 to 6? of 6 to 144?

7. Find the ratio of the following:

a. 5 to 25. b. 19 to 152. c. \$17 to \$8.50.

d. 47 to 7. e. 23 to 299. f. \$93 to \$31.

g. 83 to 7. h. 147 to 21. i. 14 bu. to 2 pk.

j. 187 to 11. k. 28 to 35. l. 24 lb. to 12 oz.

m. 244 to 2. n. 1000 to 5. o. 2 T. to 3 cwt.

p. 99 to 3. q. 40 to 1000. r. 2 cu. in. to 1 cu. ft.

RATIO

Two numbers having the same unit may be compared with each other in two ways:

By considering how much greater or less one is than the other; and

By considering how many times one contains the other.

Ratio is the relation which one number bears to another with respect to magnitude, when the numbers are compared by considering, not how much greater or less one is than the other, but how many times or parts of a time one contains the other.

The ratio of two numbers is the quotient arising from the division of one by the other.

The ratio of 18 to 6 is 3, since $18 \div 6 = 3$.

The ratio of 7 to 21 is $\frac{1}{3}$, since $7 \div 21 = \frac{7}{21} = \frac{1}{3}$.

Since ratio is a quotient, and since we have three ways of indicating division, it follows that we have three ways of expressing the ratio of one number to the other.

The ratio of 9 to 4 is expressed by $9 \div 4$, by $\frac{9}{4}$, and by $9:4$. The ratio of 7 to 13 is indicated by $7 \div 13$, by $\frac{7}{13}$, and by $7:13$.

Ratio can exist only between numbers of the same kind.

No comparison with respect to magnitude can be made between 6 hours and 11 pounds, or between \$19 and 16 miles.

The two given numbers which constitute the ratio are called the **terms** of the ratio; when spoken of together they are called a **couplet**.

The first term of a couplet is called the **antecedent**; the last term, the **consequent**.

Read: *a.* $5:14$. *b.* $21:6$. *c.* $100:1$.

Name the *antecedents* and the *consequents*.

PROPORTION

Proportion is an equality of **ratios**.

The ratios $15:3$ and $25:5$ constitute a proportion, since,
 $15:3 = 25:5$. $\frac{15}{3} = 5$. $\frac{25}{5} = 5$.

The terms of the two couplets are called **proportionals**.

Proportion may be expressed in two ways:

By placing the sign of equality, $=$, between the ratios ;
 and

By placing four dots, thus, $::$, between the two ratios.

We may express the proportion existing between 15, 3, 25, and 5 by $15:3 = 25:5$; or by $15:3::25:5$.

1. Read: *a.* $7:4 = 21:12$. *b.* $9:15::6:10$.

a. Seven to four equals twenty-one to twelve.

b. Nine is to fifteen as six is to ten.

2. Read :

a. $12 \text{ qt.} : 18 \text{ qt.} = 18 \text{ qt.} : 27 \text{ qt.}$ *b.* $3:50::12:200$.

c. $100 \text{ mi.} : 33\frac{1}{3} \text{ mi.} :: 18:6$. *d.* $x:5 = 2:10$.

A ratio consists of two terms, an antecedent and a consequent. A proportion has two couplets, or four terms.

The two outer terms of a proportion are called the **extremes**, and the two intermediate ones, the **means**.

In the proportion $3:17 = 21:119$, 3 and 119 are the *extremes*, and 17 and 21 are the *means*.

Tell the antecedents and consequents, the extremes and the means, in these proportions:

1. $1:2 = 2:4$. 4. $2:3 = 4:6$. 7. $3:4 = 6:8$.

2. $1:3 = 2:6$. 5. $2:4 = 4:8$. 8. $3:5 = 6:10$.

3. $1:4 = 2:8$. 6. $2:5 = 4:10$. 9. $3:6 = 4:8$.

In any *proportion*, the product of the *extremes* is equal to the product of the *means*.

$$6:11=18:33. \quad \text{Then } 6 \times 33 = 11 \times 18. \quad \frac{18}{33} = \frac{6}{11}.$$

$$6:11 = \frac{6}{11} \text{ and } 18:33 = \frac{18}{33}.$$

Reducing the ratios to a common denominator, we have,

$$\frac{6 \times 33}{11 \times 33} = \frac{18 \times 11}{33 \times 11}.$$

But since the fractions are equal and have the same denominators, their numerators must be equal:

$$6 \times 33 = 198 = 18 \times 11$$

But 6 and 33 are the *extremes* and 18 and 11 are the *means*; therefore, the product of the means equals the product of the extremes.

A proportion is an equation

$$4:9=20:45. \quad \frac{4}{9} = \frac{20}{45}$$

If in a proportion any term is lacking, it may be found.

1. If the product of the means be divided by one extreme, the quotient will be the other extreme.

2. If the product of the extremes be divided by one mean, the quotient will be the other mean.

$$\text{1st term} = \frac{\text{2d term} \times \text{3d term}}{\text{4th term}}$$

$$3. \quad x:10=6:15.$$

$$\text{2d term} = \frac{\text{1st term} \times \text{4th term}}{\text{3d term}}$$

$$4. \quad x:3=30:6.$$

$$\text{3d term} = \frac{\text{1st term} \times \text{4th term}}{\text{2d term}}$$

$$5. \quad 2:x=22:99.$$

$$\text{4th term} = \frac{\text{2d term} \times \text{3d term}}{\text{1st term}}$$

$$6. \quad 16:x=64:12.$$

$$7. \quad 2:15=x:90.$$

$$8. \quad \frac{1}{2}:6=x:3.$$

$$9. \quad 5:7=15:x. \quad 11. \quad x:\frac{2}{7}=\frac{7}{8}:\frac{3}{4}.$$

$$13. \quad \frac{5}{16}:\frac{1}{4}=15:x.$$

$$10. \quad \frac{1}{5}:3=7:x. \quad 12. \quad 5\frac{1}{2}:x=16:32. \quad 14. \quad 648:243=24:x.$$

PROPORTION

Proportion is sometimes called the **rule of three**, because when three terms are given, by means of them the fourth term may be found.

1. If 16 bbl. of flour cost \$112, what will 129 bbl. cost ?

$$\begin{array}{cc} \text{Barrels} & \text{Dollars} \\ 16 : 129 = 112 : x. & x = \frac{129 \times 112}{16} = \$903. \end{array}$$

Proof: Write 903 in the fourth place instead of x :

$$16 : 129 = 112 : 903.$$

$$\text{Extremes : } 16 \times 903 = 14,448.$$

$$\text{Means : } 129 \times 112 = 14,448.$$

2. If a man can walk 155 miles in 12 da., how many miles can he walk in 60 da.?

$$\begin{array}{cc} \text{Days} & \text{Miles} \\ 12 : 60 = 155 : x. & x = \frac{60 \times 155}{12} = 775 \text{ mi.} \end{array}$$

Proof: $12 \times 775 = 9300 = 60 \times 155.$

3. If 11 baskets of peaches cost \$13.42, what will 87 baskets cost ?

4. If 28 cords of wood cost \$266, what will 25 cords cost ?

5. If a man receives \$29.20 for 16 days' work, for how many days' work should he receive \$83.60 ?

6. If 16 bu. of potatoes are sold for \$12.80, what is the value of 156 bu.?

7. If a stick 7 ft. long casts a shadow of 5 ft., what is the height of a flagpole which casts a shadow of 112 ft. at the same time ?

8. If a stack of hay will feed 27 cows for 99 da., how long will it feed 55 cows ?

In solving problems by the method of proportion, think out very carefully how to state the proportion.

1. If a person can finish a journey in 100 da., traveling 10 hr. per day, how many days would he take if he traveled only 6 hr. per day?

2. What is the cost of 127 acres of land, if 871 acres cost \$8671.40?

3. If a ship has water sufficient to last a crew of 35 men for 6 mo., how long will it last a crew of 20 men?

4. If the railway fare for 100 miles is \$3.75, what should it be for 63 mi.?

5. If 16 bbl. of flour can be bought for \$97.80, what should be paid for 27 bbl.?

6. What will 65 lb. of cheese cost if \$1.30 pays for 13 lb.?

7. If 3 men build 73 rd. of fencing in 2 da., how long will they take to build 803 rd.?

8. If 176 pails of maple sap make 100 lb. of sugar, how much sugar will 1128 pails make?

9. If it costs \$20.88 to buy 108 yd. of percale, how much must be paid for 465 yd.?

10. If \$16 pay for the freightage of 720 barrels of flour, for how many barrels will \$1278 pay the freightage?

11. If 11 men plow 165 acres in a week, how many acres will 3 men plow in the same time?

12. If 4 barrels of flour make 250 four-pound loaves of bread, how many such loaves will 67 bbl. make?

13. If 190 bu. of apples make 16 bbl. of vinegar, how many barrels of vinegar will 38 bu. of apples make?

14. If 90 men can build a wall in 12 da., how many men can build it in 15 da.?

REVIEW

1. If 1 acre produces 46 bushels of oats, how many bushels will 358 such acres produce?

2. How many pounds of iron are there in the rails of a railroad track 64 miles long, when 1760 yards make a mile, and every yard of iron rail weighs 65 pounds?

3. If the average mill consumed 428 tons of coal annually, what would be the amount of coal consumed annually in a country containing 15,769 mills?

4. In a building there are 75 rooms, and each room has 4 windows. In each window there are 12 panes of glass, and each pane contains 120 square inches. How many square inches of glass are there in the building?

5. The earth, in its annual revolution, moves 19 miles a second. How far does the earth move in 1 week?

6. One Atlantic cable is composed of 7 copper wires, inclosed in gutta-percha, surrounded by 18 bundles of iron wire, each bundle composed of 7 wires. How many wires are there in the cable?

7. A railroad 95 miles long cost \$3,300,000. What was the cost per mile?

8. The Ladysmith garrison had a million days' rations when the siege began. How many days would these last 12,000 soldiers?

9. If 22 bricks make 1 cu. ft. of brickwork, how many cu. ft. are there in a wall which requires 100,000 bricks?

10. Find the value of :

a. $\frac{10}{21}$ of $\frac{35}{36}$.

b. $\frac{3}{4}$ of $\frac{1}{2}$ ft.

c. $\frac{1}{6}$ of $\frac{3}{5}$ hr.

d. $\frac{15}{44}$ of $\frac{44}{15}$.

e. $\frac{4}{5}$ of $\frac{15}{16}$ lb.

f. $\frac{6}{35}$ of $\frac{14}{15}$ T.

g. $\frac{33}{55}$ of $\frac{42}{65}$.

h. $\frac{5}{12}$ of $\frac{3}{4}$ yd.

i. $\frac{8}{11}$ of $\frac{20}{21}$ of $\frac{35}{48}$.

11. In a school there are 150 scholars, 60 of whom are girls. How many boys are there in the school?

12. A man bought a lot for \$900, and sold it for \$1300. How much did he make by the sale?

13. How many pounds of butter at 19¢ per pound would equal the value of a cow, whose price is \$47.50?

14. A man owing \$208, pays \$88. How much money does he still owe?

15. A boy going to market with 80¢, bought 40¢ worth of sugar and 30¢ worth of butter. What coin or coins may he have had left?

16. If a ship sails 250 miles a day, in how many days would it sail 10,000 miles?

17. For \$2032 how many yards of silk at \$4 a yard can be bought?

18. Divide :

a. $\frac{21}{4}$ by 7. b. $\frac{7}{19}$ by 14. c. $\frac{3}{4}$ by 9. d. $\frac{8}{38}$ by 4.

19. John Anderson owns $\frac{5}{7}$ of a railroad share worth \$117. He divides it equally among his five children. What part of a share does each child receive?

20. To the sum of 6 and 9 add the difference between 11 and 8.

21. To 13 add the sum of 6 and 5, and from the result take 8. Divide the remainder by 4.

22. From the difference between 7 and 2 take the difference between 13 and 11.

23. A man owned $\frac{3}{5}$ of a ship, and sold $\frac{1}{2}$ of his share. What share of the ship did he sell?

24. If a bushel of barley will buy $\frac{7}{10}$ bu. of wheat, how much wheat can be bought for $\frac{1}{4}$ bu. of barley?

25. The circumference of the earth is 25,000 miles; if a railroad train moves at the rate of 1250 miles a day, how many days would it require to go around the earth?

CIRCULAR MEASURE

Circular or angular measure is used to measure angles and the circumferences of circles.

A **circle** is a surface bounded by a curved line which is everywhere equally distant from a point within called the **center**. The perimeter or boundary line is called the **circumference** of the circle.

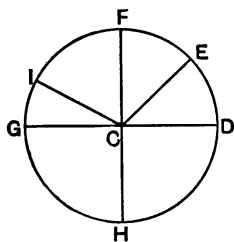
The distance from the center of a circle to the circumference is called the **radius** of the circle.

If from the vertex of an angle, as a center, we should draw a circumference, some portion of that circumference would be included between the sides of the angle. The larger the angle is, the larger will be the arc included between its sides, and the smaller the angle is the smaller will be the included arc.

Any portion of the circumference is called an **arc**.

Since the angle and the arc thus vary with each other, the arc is taken as the measure of the angle.

Draw angles and arcs and cut out paper to show this.



In this figure let C be the center of the circle and the vertex of the several angles. Let FH and CD be lines perpendicular to each other, and CE and CI be any other radii. Then the arc ED is the measure of the angle ECD ; the arc DF , of the angle DCF ; the arc DI , of the angle DCI ; the arc EF , of the angle ECF , etc.

The line from a point on one side of a circle through the center to a point on the opposite side is called the **diameter** of the circle.

The circumference of any circle, may be divided into 360 equal parts, called **degrees**.

A degree is the 360th part of the circumference of a circle. Its actual length and that of its subdivisions vary with the size of the circle.

Since FH and GD are perpendicular to each other, the angles FCD , DCH , HCG , and GCF are all right angles, and each of them must include $\frac{1}{4}$ of the angular space about the point C . The arc included between the sides of a right angle equals $\frac{1}{4}$ of the circumference, and the measure of a right angle is $\frac{1}{4}$ of 360 degrees, or 90° .

The arc of an angle of 90° is called a **quadrant**.

Four quadrants make a circle. $90^\circ \times 4 = 360^\circ$.

Quadrant is the name both of the arc of 90° and of the surface included between the *arc* and the *angle* of 90° .

An *acute* angle is always less than 90° .

An *obtuse* angle is always more than 90° .

1. What part of a circumference is an angle of 45° ?
2. What part of a quadrant is an angle of 60° ?
3. Add together these angles:
 - a. 15° , 25° , 50° . What part does this equal?
 - b. 60° , 50° , 30° , 10° . What part of a circle is each?
 - c. 20° , 60° , 40° . What part of a circle is each?
4. Divide 90° into 3 equal angles.
5. Is an obtuse angle more or less than a quadrant?
6. Draw a circle with a diameter of 2 in.; another with a diameter of 4 in.; and another with a diameter of 6 in.
7. In each of the circles in 6 draw an angle of 30° and an angle of 150° .
8. How many degrees are there in $\frac{1}{2}$ a circumference?
Use blackboard drawings and paper-cuttings to illustrate these facts.

POWERS OF NUMBERS

When a number is multiplied by itself once, twice, three, four times, the resulting products are called the second, third, fourth **powers** of the number.

The term "square" is usually employed instead of second power. The term "cube" is usually employed instead of third power.

144 is the square of 12, because $12 \times 12 = 144$.

64 is the cube of 4, because $4 \times 4 \times 4 = 64$.

81 is the fourth power of 3, because $3 \times 3 \times 3 \times 3 = 81$.

1. Find the squares of:

15	24	40	57	69	72	87	100
114	237	625	897	371	225	634	1000
32	65	56	90	125	200	500	666

2. Find the cubes of:

9	11	25	47	68	93	257	48
10	50	75	100	200	300	400	1000

3. Find the fourth power of:

2	3	4	5	6	7	8	9
10	12	15	18	20	25	30	50

4. Find the tenth power of:

2	3	4	5	6	7	9	10
---	---	---	---	---	---	---	----

5. What do you notice about the higher powers of any number? Make a drawing to show the powers of 3.

6. Read: 6^5 , 8^4 , 25^3 , 100^9 , 10^{25} .

7. Find: 4^5 , 10^5 , 16^3 , 2^8 , 3^6 , 5^9 .

8. $4^4 \div 5^2 = ?$ Answer in decimals.

9. $2^3 \div 3^2 = ?$ Answer in common fractions.

10. $100^2 \div 5^3 = ?$ Answer in decimals.

THE EQUATION

The **first member** of an equation is the part on the left of the sign $=$. The **second member** of an equation is the part on the right of the sign $=$.

Since an equation is an expression of equality between two quantities, the following changes can be made in the members of an equation without affecting their equality:

The same or equal quantities may be added to both members.

$$15 + 3 = 3 \times 6. \quad 15 + 3 + 5 = 3 \times 6 + 5.$$

The same or equal quantities may be subtracted from both members.

$$8 \times 2 = 4 \times 4. \quad 8 \times 2 - 6 = 4 \times 4 - 6.$$

Both members may be multiplied by the same or equal quantities.

$$5 = 3 + 2. \quad 5 \times 2 = (3 + 2) \times 2.$$

Both members may be divided by the same or equal quantities.

$$10 = 8 + 2. \quad 10 \div 2 = (8 + 2) \div 2.$$

Transposition changes terms from one member of an equation to the other without changing the equality.

Every $+$ or $-$ term transposed must have its sign changed to the opposite. $8 + 3 + 1 = x + 5$. Take 5 from each member. $8 + 3 + 1 - 5 = x$.

1. Find the value of x in the equation $3x - 4 = 14$.

Adding 4 to both members, $3x - 4 + 4 = 14 + 4$.

$$3x = 18.$$

Dividing by 3, $x = 6$.

2. $x + 5 = 2(8 + 4) + 7$. What is x ?

3. $x - 5 - 3 = (2x + 4) \div 12$. What is x ?

AN UNKNOWN QUANTITY

1. If 6 be taken from twice a certain number, the remainder will be 4. Find the number.

ARITHMETICAL EQUATION

$$2 \text{ times a certain number } - 6 = 4.$$

$$2 \text{ times a certain number } = 4 + 6 = 10.$$

$$\text{The number} = 10 \div 2 = 5.$$

ALGEBRAIC EQUATION

$$\text{Let } x = \text{the number.}$$

$$2x - 6 = 4.$$

$$2x = 4 + 6 = 10.$$

$$x = 10 \div 2 = 5.$$

Discuss each step in the process.

2. A purse and the money it contains are worth \$24, and the money is worth 11 times the value of the purse.

$$\text{Let } x = \text{value of purse.}$$

$$x + 11x = 24.$$

$$12x = 24.$$

$$x = 24 \div 12 = 2.$$

Solve the following equations :

$$3. \quad 2x + 3x + 4x = 18.$$

$$4. \quad 6x + 3x + 9x = 108.$$

$$5. \quad 12x - 4x - 3x = 20.$$

$$6. \quad 9x - 3x = 18 - 6.$$

$$7. \quad 17x - 114 = 198 - 7x.$$

$$8. \quad 17 - x = 2x - 1.$$

$$9. \quad 12x - 8 = 3x - 98.$$

$$10. \quad 7x + 9 = 4x + 12.$$

$$11. \quad 6 + 4x = 3x + 9.$$

$$12. \quad x + 5 = 3x - 9.$$

$$13. \quad 24x - 49 = 19x - 14.$$

$$14. \quad 4x - 22 = 34 - 3x.$$

$$15. \quad 9x - 3(5x - 6) = -30.$$

$$16. \quad 26 - 8x = 80 - 14x.$$

$$17. \quad 4x - (6x - 35) = 5x - (3x - 7).$$

18. If 1 apple costs x ¢, what will represent the cost of 2 apples? of 3? of 5? of 7?

19. If 1 lemon costs $2x$ ¢, what will represent the cost of 2 lemons? of 3 lemons? of 5? of 6?

20. I bought a number of apples at 2¢ apiece, and as many more at 3¢ apiece. If x represents the number of apples at 2¢, what will represent their cost? What will represent the cost of the apples at 3¢ apiece?

21. I bought lemons and oranges, of each an equal number; the lemons cost 2¢, and the oranges 3¢ apiece; the cost of the whole was 30¢. How many oranges were there? How many lemons?

22. I bought 2 lemons and 3 oranges for 16¢; the price of an orange was twice that of a lemon. If x represents the cost of a lemon, what will represent the cost of an orange? What will represent the cost of 2 lemons? of 3 oranges? of 2 lemons and 3 oranges? What was the cost of each fruit?

23. Fred has 4¢, and Tom has 1¢ less than Fred. How many cents has Tom?

24. If $3x - 2$ is equal to 10, to what is $3x$ equal? If $3x$ is equal to 12, to what is x equal?

25. If $5x - 3$ is equal to 17, to what is $5x$ equal? If $5x$ is equal to 20, to what is x equal?

26. If $x + 2$ is equal to 4, to what is x equal?

27. If $x + 3$ is equal to 5, to what is x equal?

28. If $2x + 3$ is equal to 8, to what is x equal?

29. A dealer sold a horse for \$100, a gain of 25%. What did it cost him?

SUGGESTION. — Let x = the cost to the dealer.

30. A boy bought a baseball for \$1.25, which was 140% of its wholesale value. Find that value.

INTEREST

Interest is money paid by a borrower to a lender for the use of money. It is estimated as a certain per cent of the sum lent.

1. What is the interest of \$100 for 1 yr. at 6%?

The interest of \$100 for 1 yr. at 6% is $\frac{6}{100}$ of \$100 = \$6.

\$100 = principal.

.06 = rate.

\$6.00 = interest.

2. What is the interest of \$611.35 for 3 yr. at 7%?

\$611.35 = principal.

.07 = rate.

\$42.7945 = interest for 1 yr.

3

\$128.3835 = interest for 3 yr.

SIX PER CENT INTEREST

In a majority of the States the legal rate of interest is 6%. Hence it is very convenient to have a short method of computing interest at that rate.

At 6% a year the interest on \$1

for 12 months is \$.06.

" 2 mo. = $\frac{2}{12}$ of 12 mo. = $\frac{1}{6}$ of 12 mo. " .01.

" 1 mo., or 30 da. = $\frac{1}{12}$ of 12 mo. " .00 $\frac{1}{2}$ = \$.005.

" 6 da. = $\frac{1}{6}$ of 1 mo. or 30 da. " .001.

" 1 da. = $\frac{1}{6}$ of 6 da. = $\frac{1}{30}$ of 30 da. " .000 $\frac{1}{6}$.

We find the interest on \$1 for any number of months by dividing the number of months by 2. The quotient is the interest in cents.

1. What is the interest of \$1 for 8 months at 6 per cent?

$8 \div 2 = 4$. \$.04.

2. What is the interest of \$1 for 7 years 3 months at 6 per cent?

7 years 3 months = 87 months, and $87 \div 2 = 43\frac{1}{2}$, \$.435.

3. What is the interest of \$1 for 11 years 7 months at 6 per cent?

4. Find the interest at 6% on \$1 for:

- | | | | |
|----------|----------|----------------|-----------------|
| a. 6 mo. | g. 7 mo. | m. 3 yr. | s. 3 yr. 8 mo. |
| b. 3 mo. | h. 5 mo. | n. 6 mo. | t. 3 yr. 11 mo. |
| c. 9 mo. | i. 2 mo. | o. 4 yr. | u. 2 yr. 2 mo. |
| d. 1 yr. | j. 1 mo. | p. 6 mo. | v. 3 yr. 10 mo. |
| e. 2 yr. | k. 4 mo. | q. 1 yr. 6 mo. | w. 4 yr. 6 mo. |
| f. 8 mo. | l. 3 yr. | r. 2 yr. 3 mo. | x. 4 yr. 10 mo. |

5. What is the interest of \$366.20 for 3 yr. 8 mo. at 6%?

3 yr. 8 mo. = 44 mo. $44 \div 2 = 22$, int. on \$1.

$$\begin{array}{r}
 \$366.20 \\
 .22 \\
 \hline
 732\ 40 \\
 7324\ 0
 \end{array}$$

\$80.56 40, int. on \$366.20 for 3 yr. 8 mo.

6. Find the simple interest on:

- | | |
|----------------------------|------------------------------|
| a. \$197 for 1 yr. | e. \$1728 for 1 yr. 6 mo. |
| b. \$1728 for 3 yr. | f. \$16.87 for 1 yr. 8 mo. |
| c. \$69 for 2 yr. | g. \$118.15 for 2 yr. 6 mo. |
| d. \$97.16 for 1 yr. 5 mo. | h. \$789.87 for 1 yr. 11 mo. |

7. A drew his note payable to B for \$1575 at 6% a year, and paid it in 2 yr. 5 mo. How much interest was due then?

In *business* interest is usually paid on long term notes, those for one year or more, annually or semi-annually. If the interest due is not paid, interest on interest, compound interest, is estimated on debts.

INTEREST

We find the interest at 6% for any number of days by dividing the number of days by 6. The quotient will be the interest in mills.

1. What is the interest of \$1 for 18 days at 6 per cent ?

$$18 \div 6 = 3 (\text{mills}) = \$.003.$$

2. What is the interest of \$1 for 26 days at 6 per cent ?

3. Find the interest at 6% on \$1 for :

<i>a.</i> 30 da.	<i>e.</i> 24 da.	<i>i.</i> 3 da.	<i>m.</i> 28 da.
<i>b.</i> 15 da.	<i>f.</i> 23 da.	<i>j.</i> 9 da.	<i>n.</i> 27 da.
<i>c.</i> 6 da.	<i>g.</i> 18 da.	<i>k.</i> 16 da.	<i>o.</i> 21 da.
<i>d.</i> 20 da.	<i>h.</i> 12 da.	<i>l.</i> 14 da.	<i>p.</i> 10 da.

4. What is the interest of \$1 for 7 yr. 4 mo. 27 da. at 6 per cent ?

$$7 \text{ yr. } 4 \text{ mo.} = 88 \text{ mo.}, \text{ and } 88 \div 2 = 44 (\text{cents}) = \$.44 \\ = \text{interest for } 7 \text{ yr. } 4 \text{ mo.}$$

$$27 \div 6 = 4\frac{1}{2} (\text{mills}) = \$.0045 = \text{interest for } 27 \text{ da.}$$

$$$.4445 = \text{interest for } 7 \text{ yr. } 4 \text{ mo. } 27 \text{ da.}$$

5. Find the interest on \$1 at 6 per cent a year for :

<i>a.</i> 16 years 4 months.	<i>d.</i> 4 years 7 months 10 days.
<i>b.</i> 3 years 3 months 3 days.	<i>e.</i> 11 years 1 month.
<i>c.</i> 5 years 11 months.	<i>f.</i> 1 year 9 months 25 days.

We can find the interest at 6% on any sum of money by finding the interest on \$1 for the period and multiplying this by the number of dollars in the given principal.

6. Find the interest at 6% on :

<i>a.</i> \$250 for 3 mo. 10 da.	<i>c.</i> \$560 for 8 mo.
<i>b.</i> \$175 for 4 mo. 20 da.	<i>d.</i> \$1000 for 1 yr. 2 mo. 3 da.
<i>e.</i> \$5,000,000 for 1 yr. 3 mo. 18 da.	

QUESTIONS

1. Give the multiplication tables, 2 to 12.
2. What are the three dimensions of a *solid*?
3. What is a *cube*? a *circle*? a *triangle*?
4. Discuss *area*.
5. How thick is a pane of glass?
6. How high is an ordinary flour barrel?
7. What is the height of a horse?
8. What is meant by *perpendicular*?
9. What is an ordinary height for a ceiling in a house?
for a chair?
10. What is an ordinary height for men?
11. How far can a boy jump who is a good jumper?
12. In what time can a good runner run a mile?
13. Of what use to boys and girls are the facts of
weights and measures?
14. What amounts of money can boys and girls earn at
different kinds of work?
15. What amounts of money do men earn?
16. What do we mean when we say that a man is a
millionaire? Explain.
17. What is *property*? Why do we measure property
by *money*? What is money?
18. How high is an oak? a flag pole? a telegraph pole?
19. What do you know about the heights of *mountains*?
depths of *mines*? areas of *farms*? widths of *rivers*?
20. How long is a railroad car? a locomotive engine?
21. Are houses worth more than factories? Discuss.
22. Of what use to a boy or a girl is Arithmetic?
23. What are some of the uses of Arithmetic to men?

REVIEW

1. A prize captured by two United States cruisers in the Spanish-American war was valued at one hundred eighty-four thousand seven hundred seventy-five dollars. If this sum had been divided equally among the four hundred seventy-five men, composing the crews of both vessels, what would have been the share of each man?

2. What is the ninth part of \$972?

3. A canal 126 miles in length cost \$3,860,000. What was the cost per mile?

4. Find $\frac{3}{4}$ of $\frac{8}{5}$ of $\frac{24}{5}$.

5. A cubic foot of distilled water weighs 1000 ounces. What is the weight of one cubic inch?

6. If one man can do a piece of work in 504 days, how many days will it take 24 men to do the same work?

7. If an iceberg moves 6 miles a day, how many days will it take to move 2556 miles?

8. If in 178 books there are 56,070 pages, how many pages are there on an average in a book?

9. If 37 barrels of apples are sold for \$55.50, what is the cost of a barrel?

10. Three partners began business with a capital of \$1793; the first had \$714, the second had \$417. What did the third have?

11. After selling 1,016,085 oranges a merchant had two lots of oranges left, one lot containing 85,619 oranges, and the other 89,756. How many oranges had he at first?

12. Divide:

a. $\frac{24}{7}$ by 5.

c. $\frac{75}{98}$ by 15.

e. $\frac{88}{103}$ by 11.

b. $\frac{7}{9}$ by 12.

d. $\frac{11}{12}$ by 6.

f. $\frac{450}{33}$ by 75.

13. Divide :

- a. $\frac{9}{16}$ by 6. c. $\frac{24}{47}$ by 16. e. $\frac{1220}{2277}$ by 9.
 b. $\frac{1728}{1848}$ by 36. d. $\frac{18}{55}$ by 12. f. $\frac{144}{155}$ by 60.

14. Divide :

- a. 7 by $\frac{4}{5}$. b. 14 by $\frac{7}{5}$. c. 36 by $\frac{3}{4}$. d. 81 by $\frac{9}{10}$.

15. A man bought a million bricks. He used 8609 in building a wall, 87,609 in building a house, and 20,760 in building a stable. How many did he have left?

16. A merchant paid \$12.50 for figs, \$17.35 for raisins, \$29 $\frac{1}{8}$ for spice, \$164.95 for sugar, \$716.29 for tea, and \$690 for coffee. He had \$2700 before paying for his goods. How much money had he left?

17. A family's expenses for one month were: baker's bill, \$5.73; butcher's bill, \$10.91; groceries, \$12.75; fruit, \$3.29; rent, \$26.25; servant's wages, \$16; tailor's account, \$17.87; shoemaker's bill, \$11.63; coal, \$5.75; and sundries, \$19.47. What was the total account for the month?

18. Subtract :

- | | | | | | |
|--------------|--------------|--------------|--------------|--------------|-------------|
| a. 9864 | b. 7426 | c. 6239 | d. 4729 | e. 6258 | f. 4625 |
| <u>3712</u> | <u>3818</u> | <u>4127</u> | <u>501</u> | <u>36</u> | <u>1846</u> |
| g. 13602 | h. 20607 | i. 30102 | j. 31607 | k. 40102 | |
| <u>11795</u> | <u>19089</u> | <u>18607</u> | <u>19089</u> | <u>19007</u> | |

19. Add the numbers in 18.

20. Draw a circle with arcs of these degrees: 60°, 30°, 45°, 180°.

What fractional part of the circumference is each arc?

21. Find the prime factors in: 865, 939, 1246.

22. Draw a parallelogram, and measure it.

23. Draw a triangle, and measure it.

24. Estimate their areas by such scales as:

$$\frac{1}{8} \text{ in.} = 1 \text{ ft.} \quad \frac{1}{16} \text{ in.} = 1 \text{ yd.} \quad \frac{1}{4} \text{ in.} = 1 \text{ mi.}$$

REVIEW

1. Add each combination. Subtract. Multiply.

13	15	14	13	12	11	12	16	11
<u>5</u>	<u>7</u>	<u>7</u>	<u>7</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>7</u>	<u>4</u>
15	14	10	16	12	12	14	12	17
<u>7</u>	<u>5</u>	<u>3</u>	<u>8</u>	<u>9</u>	<u>8</u>	<u>5</u>	<u>2</u>	<u>8</u>
13	11	10	12	17	13	11	17	10
<u>5</u>	<u>4</u>	<u>8</u>	<u>7</u>	<u>9</u>	<u>8</u>	<u>9</u>	<u>7</u>	<u>6</u>
13	11	15	14	13	11	13	14	10
<u>8</u>	<u>3</u>	<u>8</u>	<u>7</u>	<u>5</u>	<u>2</u>	<u>8</u>	<u>6</u>	<u>7</u>
13	12	15	11	13	10	14	12	16
<u>6</u>	<u>5</u>	<u>8</u>	<u>2</u>	<u>7</u>	<u>9</u>	<u>5</u>	<u>3</u>	<u>8</u>
13	11	12	14	10	13	17	13	15
<u>8</u>	<u>6</u>	<u>7</u>	<u>9</u>	<u>2</u>	<u>6</u>	<u>9</u>	<u>8</u>	<u>1</u>
10	13	14	17	11	12	16	12	14
<u>6</u>	<u>9</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>5</u>	<u>9</u>
14	16	11	10	13	17	11	10	18
<u>8</u>	<u>9</u>	<u>5</u>	<u>6</u>	<u>4</u>	<u>9</u>	<u>7</u>	<u>4</u>	<u>9</u>
10	14	15	12	18	15	14	10	13
<u>6</u>	<u>9</u>	<u>7</u>	<u>6</u>	<u>9</u>	<u>6</u>	<u>8</u>	<u>5</u>	<u>4</u>

2. In these combinations divide each larger number by the smaller. What fraction do we get in each case by calling the smaller number the numerator, the larger the divisor?

3. If meat is $12\frac{1}{2}$ cents per lb., what will 6 lb. cost? 10 lb?

4. If 1 yd. of braid is worth 12¢, what are $4\frac{1}{2}$ yd. worth?

5. When rice is sold for 9¢ a pound, what must be paid for $7\frac{1}{2}$ pounds? for $8\frac{1}{2}$ pounds? for $9\frac{1}{2}$ pounds?

6. In the following examples each cost price is to be multiplied by each rate of profit or loss:

Cost Price.	Rate of Profit or Loss.
I \$100.	a. 5% loss.
II \$500.	b. 10% profit.
III \$20.	c. 20% loss.
IV \$350.	d. 30% profit.
V \$120.	e. 5% profit.

Find the total gain or loss.

7. If a boy earns \$.875 in 1 day, how much will he earn in $9\frac{1}{4}$ days?

8. If one A. of land cost \$48, what will $\frac{5}{8}$ A. cost?

9. I owe one man \$245, and another man \$325, a third man \$187, and a fourth man \$96. How much do I owe the four men?

10. If a yard of cloth cost \$1 $\frac{5}{8}$, what will $9\frac{1}{4}$ yd. cost? $\frac{3}{4}$ yd.? 3 yd.? $12\frac{1}{2}$ yd.?

11. Multiply:

a. .23 by .009. b. 26.087 by .408. c. 341.45 by .007.
 d. 7.02 by 5.27. e. .3045 by .00061. f. .3020 by .015.
 g. .004 by .04. h. 6.23 by .049. i. .132 by .241.

12. If $\frac{5}{8}$ of a bbl. of apples cost \$1 $\frac{1}{2}$, what will a bbl. cost? 2 bbl.? 10 bbl.?

13. The length of the Danube is about 1750 mi.; of the Dnieper, 1250 mi.; of the Don, 1120 mi.; of the Rhine, 700 mi.; of the Elbe, 800 mi.; and of the Rhone, 650 mi. What is their aggregate length?

14. The length of the Amazon River is 3600 mi.; of the Missouri-Mississippi, 5500 mi.; of the Rio de la Plata, 2240 mi.; of the MacKenzie, 2000 mi.; of the St. Lawrence, 2100 mi.; of the Orinoco, 1600 mi.; and of the Rio Grande, 2150 mi. What is their aggregate length?

REVIEW

1. A grocer spent \$56 $\frac{2}{3}$ for canned fruits, giving \$1 $\frac{1}{2}$ a dozen cans. How many dozen cans did he buy?

2. Multiply sixty-five thousandths by two hundred and twenty thousandths.

3. A roller 4.12 ft. in circumference makes 35.2 revolutions in passing from one end of a lawn to another. What is the length of the lawn?

4. From 1 A. of land I sold two house lots, each 100 sq. ft. What is the value of the rest of the land at 8¢ per sq. ft.?

5. How much does $.06896 \times 1.45$ differ from 1?

$.00674 \times 321?$ $.095 \times 1000?$ $25.238 \times 12.17?$

$.23 \times .25?$ $.00514 \times 867.59?$ $4.285 \times 17.05?$

6. If 1 ton of coal cost \$4.275, what will 300 tons cost?

7. In what time will 12 horses consume a stack of hay that will last 21 horses 6 $\frac{2}{3}$ wk.?

8. A dealer sold 372 lb. of raisins at 15¢ per lb., 434 lb. of currants at 12 $\frac{1}{2}$ ¢ per lb., and 16 lb. of coffee at \$.33 per lb. What was the amount of these sales?

9. How many gallons are there in a cubic foot? There are 231 cu. in. in 1 gal.

10. Find the quotients:

- | | | |
|----------------------|----------------------|------------------------|
| a. $33000 \div 16.$ | b. $12300 \div 25.$ | c. $5439 \div 49.$ |
| d. $12771 \div 27.$ | e. $25830 \div 35.$ | f. $34104 \div 56.$ |
| g. $452160 \div 64.$ | h. $388575 \div 55.$ | i. $2953216 \div 128.$ |

11. Write ten numbers to be added together, using billions, trillions, and quadrillions.

12. Subtract two numbers, one in decillions, the other in octillions.

13. Write a very, very small fraction in decimals.

14. I lost 10 per cent of \$975. How much had I left?
15. Find : *a.* $46109 \div 49$. *d.* $135791 \div 125$.
b. $100000 \div 165$. *e.* $479102 \div 165$.
c. $975319 \div 616$. *f.* $63844 \div 135$.
16. What will 79 bu. of wheat cost at $\$ \frac{7}{8}$ per bu.?
17. Find the cost of $4\frac{3}{8}$ bu. of peaches at \$1.75 per bu.
18. Find the cost of 7 tons of hay at $\$23\frac{3}{8}$ per ton.
19. If a man receives $\frac{3}{8}$ of a dollar for one hour's labor, what will he receive for 21 hours' labor?
20. Reduce to common denominator and add :
a. $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{7}$. *e.* $\frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}$.
b. $\frac{5}{8}, \frac{5}{8}, \frac{2}{9}, \frac{13}{14}$. *f.* $\frac{1}{10}, \frac{5}{16}, \frac{7}{24}, \frac{8}{35}, \frac{9}{40}, \frac{11}{48}$.
c. $\frac{1}{12}, \frac{9}{16}, \frac{13}{22}, \frac{2}{9}, \frac{13}{18}, \frac{7}{20}$. *g.* $\frac{3}{8}, \frac{5}{24}, \frac{7}{10}, \frac{5}{32}, \frac{11}{48}$.
d. $\frac{1}{5}, \frac{2}{3}, \frac{7}{10}, \frac{5}{6}$. *h.* $\frac{9}{16}, \frac{8}{25}, \frac{33}{50}, \frac{23}{48}, \frac{11}{32}, \frac{1}{240}$.
21. One variety of Egyptian granite weighs 350 lb. per cu. ft. What is the weight of 1 cu. yd. of such granite?
22. I sent to Liverpool 5000 barrels of apples, which cost me \$1.25 per barrel. 25 per cent of the apples were thrown overboard in a storm, and the remainder were sold at \$2 per barrel. What was gained?
23. If 55 men earn a certain sum of money in 21 days, how many men can earn the same sum in 77 days?
24. How many days, at \$1.50 a day, must 48 men work in order to pay for 240 bushels of potatoes at 60 ¢ a bushel?
25. A man worked for 20 days at \$2.40 a day, and took in payment 60 bushels of wheat. How much was the wheat valued at a bushel?
26. $x : 5 = 30 : 20$. 27. $2 : x = 24 : 9$.
28. What is the area of a rectangle 3 ft. by 10 yd.?

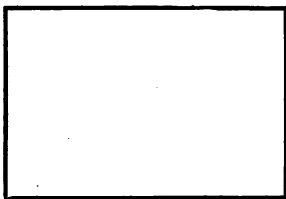
SURFACES

A **plane** has two dimensions, **length** and **breadth**.

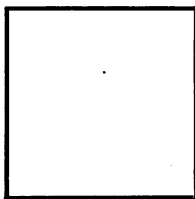
A four-sided plane figure, with all its angles right angles, is called a **rectangle**.

A rectangle, with all its sides equal, is called a **square**.
A square has four equal sides and four equal angles.

RECTANGLE



SQUARE



If a square measures a foot on each side, it is called a square foot; if it measures a yard on each side, it is called a square yard. A square unit is any surface equivalent to a square one unit long and one unit wide.

All surfaces are measured by **square measure**, that is, by the number of squares of a given size to which the surfaces are equivalent.

The sum of the lengths of all the sides of any figure is its **perimeter**.

1. Measure the lengths of the perimeters of the two figures on this page.

2. Measure the length of the sides of the cover of this book.

3. How many sq. in. are in a square whose perimeter is 16 in. long? Draw the square and prove the answer.

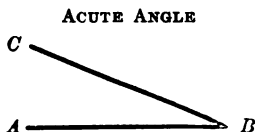
4. How many sq. in. are in a rectangle whose perimeter is 24 in. long? Draw the figure on the blackboard.

SURFACE MEASURE

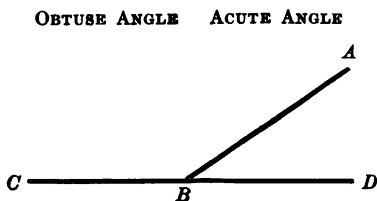
When two lines meet at a point, their difference in direction is called the **angle** of the two lines. The point where they meet is called the **vertex** of the angle.

The lines BA and BC form an angle whose vertex is at B .

In reading an angle, the letter at the vertex is always made the middle one. The angle may be read either as the angle ABC or as the angle CBA .

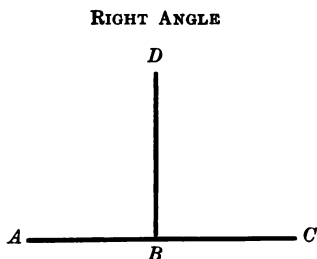


In this figure there are two angles, DBA and ABC . The first is the difference in direction of the two lines AB and BD , and the second is the difference of the two lines AB and BC . Which angle is the larger?



When the two angles formed by one straight line meeting another straight line are equal to each other, they are called **right angles**, and the two lines are said to be **perpendicular** to each other.

When a straight line BD meets a straight line AC so as to make the **adjacent** angles ABD and BDC equal to each other, then ABD and DBC are each right angles, and DB and AC perpendicular to each other.



An angle greater than a right angle is called an **obtuse** angle, and one less than a right angle is called an **acute** angle.

REVIEW

1. Give the Tables of Measures.
2. Give the aliquot parts of one hundred.
3. From 1 take .125.
4. From 63.25 take 50.
5. What are the factors of 18? of 32? of 144? of 150?
6. A farmer who had 80 sheep sold 20% of them.
How many did he sell?
7. A man having \$4000 invests 25 per cent of it.
What sum does he invest?
8. What part of an area 3 yd. square is 3 sq. yd.?
9. If $\frac{5}{8}$ T. of coal cost \$3.20, what will $\frac{1}{8}$ T. cost?
10. Find the cost of 5 tons of coal at \$3 $\frac{1}{2}$ per ton.
11. If 2 dozen pears cost 50¢, what will 3 dozen cost?
12. How many bushels of corn at 60¢ a bushel can be exchanged for 12 bushels of potatoes at 55¢ a bushel?
13. I sold a horse for \$300, gaining thereby \$40.75.
What did the horse cost me?
14. I bought a bicycle for \$50, and sold it for 5% more than it cost me. How much did I gain?
15. If 6 men can build a boat in 120 da., how long will it take 24 men to build it?
16. If $\frac{1}{4}$ of $\frac{3}{4}$ of the cost of a city hall was \$300,000, what was the whole cost?
17. If 7 men can pave a street in 21 da., how many men will be required to pave it in 3 da.?
18. A room is 12 ft. long, 12 ft. high, and 10 ft. wide.
How many cubic feet are there in the room?
19. I bought grain for \$425, and sold it at 100 per cent profit. What was my gain?
20. Draw on the blackboard a decagon, and divide it into ten equal triangles.

21. From 8960.320507 take 63.001.
22. From 75 hundredths take 75 thousandths.
23. Find the value of :
- a. $37.5 + 48.26 + 0.00831 - 85.759$.
- b. $2.02 - 0.0909 - 1.9009 + 19.009 - 9.029209$.
24. A piece of cloth measured $23\frac{5}{8}$ yd. before fulling, but only $21\frac{5}{8}$ yd. after it. How much did the cloth shrink ?
25. A dealer bought 1500 bbl. of apples, and found 12% of the fruit was spoiled. How many bbl. did he lose ?
26. Increase \$225 by 8 % of itself.
27. Reduce to common fractions :
- a. .75. c. .375. e. .4. g. .625. i. .21875.
- b. .625. d. .415. f. .625 h. .008. j. .024.
28. What is 220% of 117 barrels of apples ?
29. Reduce the fractions $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{5}{7}$ to equivalent fractions having a common denominator.
30. Decrease \$360 by 6 % of itself.
31. Two persons have \$24 between them ; the first has three times as much as the second. How much has each ?
32. Three boys are to share 24¢ among them ; the second boy is to have twice as many as the first, and the third boy is to have three times as many as the first.
- Let x represent the share of the first.
33. Wilson agreed to sell $37\frac{1}{2}$ cords of wood to Jackson. He delivered $9\frac{1}{4}$ cords one week, $8\frac{7}{8}$ the next, and $10\frac{5}{8}$ the next. How many cords then had he still to deliver ?
34. Draw a trapezoid, giving its measures. Divide it into triangles, using dotted lines. Find its area.
35. What is the circumference of a circle with radius 2.3 in. ? What is its diameter ?

REVIEW

1. The greater of two numbers is equal to six times the smaller, and their sum is 49. Find the numbers.

2. Telegraph poles were placed 132 feet apart. How many feet were there between the first pole and the last of 236 such poles?

3. If a man wastes 6¢ a day, how much money will he waste in thirty years?

4. $\frac{13}{18} - \frac{8}{15} + \frac{11}{20} - \frac{13}{30} = ?$

5. Multiply 818,647,362 by 2, 3, 4, 5, 6, 7, 8, 9.

6. At $7\frac{1}{2}$ ¢ per yd., what will 43 yd. of braid cost?

7. If a gallon of kerosene burns $4\frac{5}{8}$ hours in a lamp, how long would 15 gallons burn?

8. I spent \$3.62 $\frac{1}{2}$ for a vest; \$1 $\frac{3}{4}$ for a pocket-book; \$1 $\frac{3}{8}$ for a knife; \$.62 $\frac{1}{2}$ for a necktie; \$.87 $\frac{1}{2}$ for a key ring. How much did I spend in all?

9. Two trains start at the same point and travel in opposite directions, one at the rate of 34 miles an hour, the other at the rate of 28 miles an hour. How far apart will the trains be at the end of 14 hours?

10. If a man borrows \$800 at 6% for 15 mo., at the end of the time how much money will he owe?

11. $\frac{15 \times 32 \times 84 + 3}{24 \times 9 \times 33 \times 60} = ?$ 12. $\frac{3 + 6 \times 5 + 15 - 7}{7 - (8 - 5) \times 2 + 2} = ?$

13. $x = 5(6 - 3) + (8 - 4)(5 + 2)$. What is x ?

14. $7:45 = 63:x$. 15. $x:9\% = \$100:\$333.33$.

16. Draw a triangle with base $3\frac{1}{2}$ in. long and altitude 2 in. Find its area.

17. Draw a circle with a radius of 2 in.

REVIEW

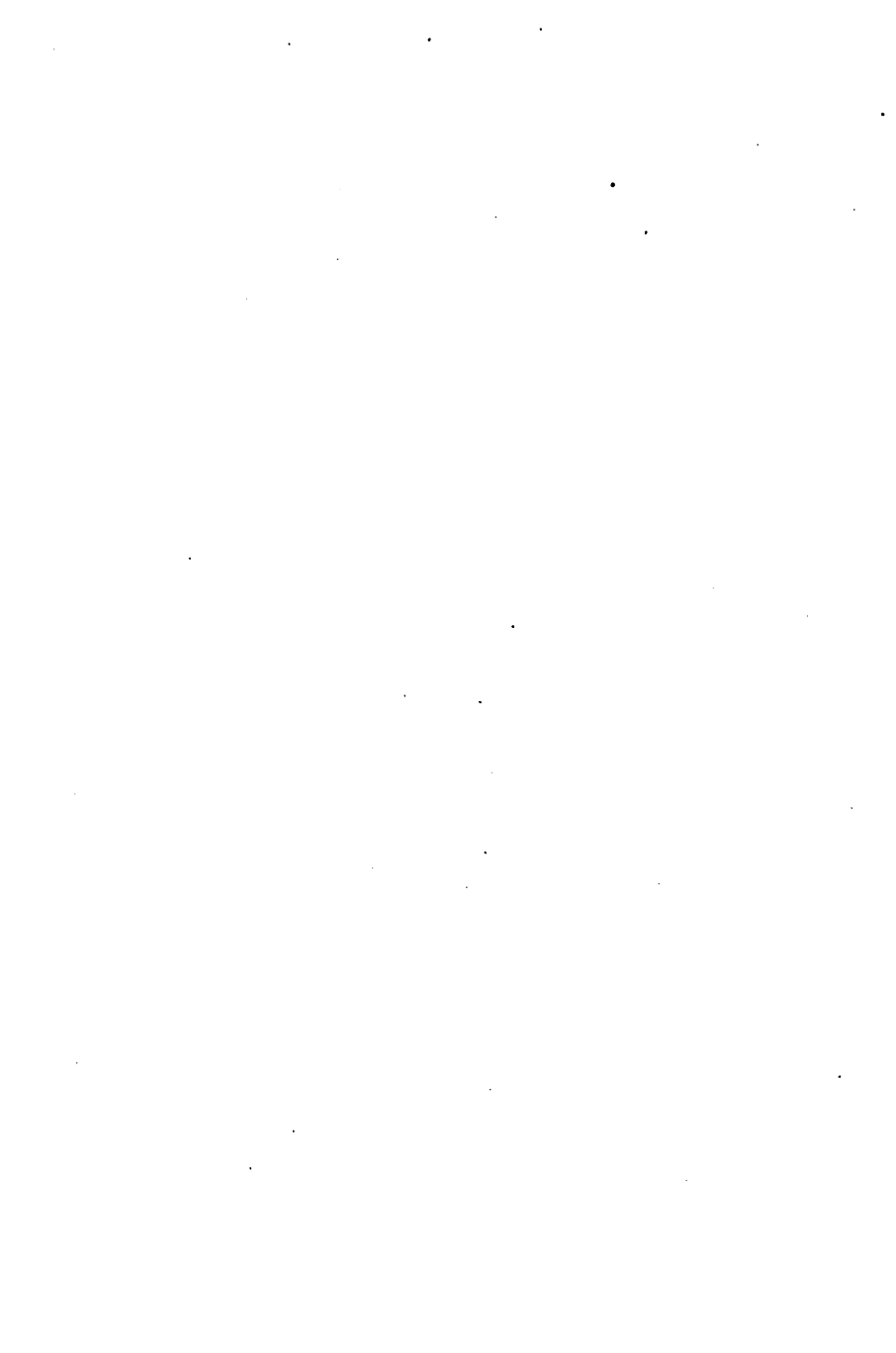
1. What is 2 per cent of \$750?
2. How many cubic feet are there in a rectangular bin 8 ft. by 5 ft. by 4 ft.?
3. If $\frac{7}{8}$ A. of land cost \$21, what will $\frac{1}{8}$ A. cost?
4. What will 56 lbs. of salt cost at $\frac{3}{4}$ ¢ per lb.?
5. How many tons of coal at \$6 a ton will pay for 30 cords of wood at \$4 a cord?
6. Harry and Tom have 75¢, but Harry has twice as many cents as Tom. How many cents has each boy?
7. If M. feet of boards cost \$12, what will 5 M. feet cost? 6 M.? 9 M.? 12 M.?
8. If 30 men can earn a certain sum of money in 25 days, how long will it take one man to earn it?
9. How many cubic feet are contained in a pile of cordwood 8 ft. 4 ft. 4 ft.?
10. What is the cost of $2\frac{1}{4}$ lb. of butter at 24¢ per lb.?
11. What are the factors of 48?
12. Find the greatest common divisor:

a. 8, 14, 12.	b. 25, 10, 20.	c. 15, 30, 35.	
d. 3, 12, 6.	e. 32, 8, 40.	f. 18, 27, and 105.	
g. 8, 12, 20.	h. 60, 70, 80.	i. 24, 12, 16.	
13. Tell the meaning of each of these terms:

parallelogram	numerator	arc	angle
obtuse angle	denominator	decimal	interest
cancellation	perimeter	quotient	radius
proportion	equation	quadrant	factor
antecedent	consequent	perpendicular	exponent
greatest common divisor		symbol of aggregation	

REVIEW

1. What part of 63 is 1890? Answer in decimals.
2. $3\frac{3}{8} + 2\frac{5}{6} - \frac{7}{12} + 3\frac{4}{9} = ?$
3. A man bought 56 head of cattle for \$25 a head, and 94 for \$32 a head. If he sells the whole lot at \$30 a head, will he gain or lose, and how much?
4. A man whose yearly salary is \$3500 expends for rent \$600; for fuel and clothing, \$570; for provisions, \$635; and for other expenses, \$1200. How much money can he save in $11\frac{5}{12}$ years, the term of a series of stock in a local building and loan association?
5. If a man can build a wall in 32 days by working 8 hours a day, how many days would he take to build the same wall, working only 6 hours a day?
6. A dealer sold $17\frac{7}{12}$ tons of cannel coal for \$187 $\frac{3}{4}$. At that rate what would $\frac{3}{4}$ of a ton cost?
7. What is the interest at 6% on \$450 for 90 days?
8. $8(3 + 4) - (10 + 5) + x = 45$. Find x .
9. Add: one hundred five ten-thousandths; one and thirty-seven hundredths; ten thousand eight hundred seventy-five millionths.
10. What kind of an angle measures 90° ? Draw it.
11. Draw a parallelogram of any form or size and measure its area, using no fraction smaller than $\frac{1}{8}$.
12. Write a bill of goods with five purchases.
13. Write a problem, using facts connected with Linear Measure.



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